THE USE OF OPERATING CHARACTERISTIC CURVES IN THE VALIDATION OF THE ASSUMPTION OF MULTIVARIATE NORMALITY AND DETERMINATION OF SAMPLE SIZE

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SUMMARY

This research addresses the problem of validating the assumption of multivariate normality. Emphasis is placed on developing a validation procedure which takes into account the problem of sample size. As a result of a literature survey, three tests for multivariate normality are selected for this investigation.

Using the power of the test as a criterion, power curve relationships for various alternative multivariate distributions are developed. From these relationships, a heuristic procedure and general guidance are given to aid in testing for multivariate normality and judging sample size adequacy.

The approach is demonstrated for an actual operational test involving multivariate data and a need to validate the assumption of multivariate normality.

The proposed approach and power curve relationships are found to be viable aids in validating the assumption of multivariate normality and determining adequate sample size.

CHAPTER I

INTRODUCTION

Background

The Army System Acquisition Process

The United States Department of Defense has an acquisition procedure which was carefully designed to assure that only the most efficient and cost effective defense systems are adopted for implementation. The United States Army has a material systems acquisition procedure which is very similar to the procedure used by the Department of Defense.

The materiel acquisition procedure of the Army is supported by the United States Army Operational Test and Evaluation Agency (OTEA). In general, OTEA is responsible for operational testing, force development testing and experimentation and joint user testing.

The types of testing discussed above are conducted prior to the acceptance of any materiel systems. Operational Testing (OT) is designed to evaluate the effectiveness of the force which uses the system under consideration; Developmental Testing (DT), on the other hand, is concerned with design aspects of the system, independent of forces which might utilize the system. Results of testing are forwarded through channels to the Army Systems Acquistion Review Council (DSARC), with the final decision of acceptance or rejection resting with the Secretary of Defense (7,17,18). Since this research is concerned to a large extent with the operational testing portion of the acquisition process, a more detailed discussion of operational testing is given below. For a more in depth discussion and summary of acquisition procedures of the Army and Department of Defense, the reader is referred to Burnette (10).

Operational Testing

As mentioned, operational testing is concerned with evaluating a system as it is employed by a military force. Thus, during the OT segment of the acquisition process, the analyst must establish some definition of operational effectiveness, which may be measured and expressed quantitatively. Such measures, referred to as Measures of Effectiveness (MOE), or Critical Issues are critical in that a poor choice of MOE could result in a system being wrongly accepted or rejected. It is assumed throughout this research that the "best" MOE's for the OT segment have been selected. For a complete discussion of the selection of MOE's the reader is referred to Williams (51).

Once MOE's have been selected, experiments are designed to facilitate calculation of MOE's. Data collected on MOE's must then be subjected to appropriate statistical analysis. It is necessary that the statistical analysis used allow analysts to compare systems. As a result, ASARC and DSARC are provided with appropriate information for decision making.

Objective, Procedure and Scope

Direction for this research was provided by a task defined by OTEA:

> The task is to apply the principles and techniques of multivariate statistical methods to assist in determining operational test design requirements. Considerations in the test design process must be given to validating the basic assumptions of multivariate normality in order to subsequently apply multivariate analysis.

Operational testing at even the most basic level is likely to involve a number of MOE's which are likely to be correlated and behave according to some multivariate distribution. Since knowledge of multivariate distribution is limited, analysis is greatly simplified when data may be assumed to be multivariate normal. However, the assumption of multivariate normality is generally made without the aid of rigorous validation procedures. Consequently, inherent in any development of multivariate methodology is the problem of validating the assumption of multivariate normality. Included in the problem is the question of whether sufficient data has been collected to adequately test for multivariate normality.

This research focuses on developing a validation procedure which is concerned with the problem of sample size for testing the normality assumption.

The power of the test is used as a criterion for comparison because of its implications for experimental design. Its flexibility as criterion allows the analyst to judge the adequacy of a particular sample. In addition, knowledge of power relationships enables the analyst to better design multivariate experiments via parameters such as sample size.

The scope of this research is concentrated on the development of operating characteristic curves and power curve relationships for selected tests of multivariate normality. A survey of the literature is conducted to determine appropriate multivariate test statistics. Alternative distributions are selected on the basis of MOE's generally used in the OTEA environment for operational testing.

Monte Carlo techniques are used to calculate estimates of critical values and powers. For critical value determination, test statistics are computed and ordered, with the critical value being the ordered test statistic corresponding to the specified significance level. Powers are computed by computing test statistics based on samples taken from multivariate distributions which are known to be non-normal and comparing these statistics to critical values with the power being equal to the number of samples rejected as not being multivariate normal divided by the number of samples taken. This research is restricted to cases where the number of variates (P) is less than or equal to eight and the sample size (N) is less than or equal to twenty. While these restrictions are necessary due to computational costs, they cover the range of P and N likely to be encountered in operational testing. Likewise, the alternative distributions used in this research (discussed in Chapter III) are restricted to those which represent cases likely to be encountered by OTEA.

CHAPTER II

REVIEW OF APPLICABLE MULTIVARIATE RESULTS AND TECHNIQUES

Introduction

This chapter contains a concise review of multivariate results and techniques which serve as a framework for this research. Specifically, tests and procedures for testing the assumption of multivariate normality are reviewed.

Development of Tests of Multivariate Normality

Practically all of the tests for multivariate normality in use today were derived as extension of their univariate counterparts.

Malkovich (30,31) conducted an extensive comparison of various tests for multivariate normality. Among the statistics studied were $\sqrt{b_1}$ (univariate skewness) and b_2 (univariate kurtosis) and the W statistic of Shapiro and Wilk (42,43). Malkovich generalized these statistics to the multivariate case. In addition, he examined several statistics that are based on the fact that if a vector of observations, $\underset{\sim}{Y}$ is $N(\mu_0,\Sigma_0)$, where $\underset{\sim}{\mu}_o$ is the mean vector and Σ_o is the covariance matrix, then $Q = (Y-\mu_o)\Sigma_o^{-1}(Y-\mu_o)$ is distributed χ^2 with P degrees of freedom if $\underset{\sim}{Y}$ is a Pxl vector. Malkovich lists these statistics as the ones proposed by Kolmogorov-Smirnov, Cramer-Von Mises along with the chi-square method and statistically equivalent blocks. He also explored several statistics based on transformations of Q to a N(0,1) random variable, which he found to be inferior in performance in a number of cases.

Malkovich concluded that the generalized Shapiro & Wilk statistic, W*, and the generalized statistics for skewness and kurtosis were generally the most powerful statistics of the numerous statistics compared. Although the statistics discussed by Malkovich for skewness and kurtosis appeared to be the most powerful in a number of instances, their computation requires use of Newton-Raphson iterations, which require costly computer runs. This becomes acutely important as the number of variates increases. As a result, Malkovich only presents results for the bivariate case (P=2). The W* statistic, on the other hand, is relatively easy to compute.

In a later work, Mardia (32) described statistics for skewness (B1,P) and kurtosis (B2,P) which are strictly multivariate in the sense that they are not generalized from the univariate case. Thus, they do not involve a search procedure over a range of possible univariate statistics as do statistics for skewness and kurtosis presented by Malkovich. In addition, Mardia presented forms of the B1,P and B2,P statistics that are comparatively easy to compute from sample data and are readily programmable. It is noted here that both Malkovich and Mardia restricted their work to a small number of cases of variates and sample sizes. Thus, it was deemed necessary, as an integral part of this research, to extend their results to a more practical range of cases from the viewpoint of OTEA. Based on computational considerations and the screening of possible statistics presented by Malkovich, the W*, B1,P and B2,P statistics were selected as test statistics for this study. A more detailed discussion of these statistics is given below.

The W* Statistic

Shapiro and Wilk (42) discussed a statistic (W) for testing univariate normality. Malkovich (31) generalized this test statistic to the multivariate case (W* statistic). A brief summary of this generalization appears below.

W Statistic (Univariate Case)

Suppose that Y_j for j=1,...,n is a random sample from a standard normal distribution and let Y(j) for j=1,...,n be the ordered observations; then the W statistic as defined by Shapiro and Wilk is given by

$$W = \frac{\left[\sum_{j=1}^{n} a_j (Y_{(j)} - Y)\right]^2}{\sum_{j=1}^{n} (Y_j - Y)^2} \tag{2.1}$$

where

$$a_j = \frac{E(Y_{(j)})}{\{\sum_{j=1}^{n} [E(Y_{(j)})]^2\}^{\frac{1}{2}}}$$

Shapiro and Wilk (42) gave methods for estimating the a_j and demonstrated the invariance property of the W statistic.

W* Statistic (Multivariate Case)

Malkovich (31) and Malkovich and Affifi (30) explained a generalization of the W statistic (univariate case) to the W* statistic (multivariate case). The procedure for calculating the W* Statistic is as follows:

1. Let Y_m be the observation vector for which

$$(Y_m-\bar{Y})'A^{-1}(Y_m-\bar{Y}) = \max\left((Y_j-\bar{Y})'A^{-1}(Y_j-\bar{Y})\right) \tag{2.2}$$

$$\text{where } A = \sum_{j=1}^{n}(Y_j-Y)(Y_j-Y)'$$

2. Order the statistics

$$UU_j = (Y_m-\bar{Y})A^{-1}(Y_j-\bar{Y}), \quad j=1,2,\ldots,n$$

3. The test statistic is

$$W^* = \frac{[\sum a_j UU_j]^2}{(Y_m-\bar{Y})A^{-1}(Y_m-\bar{Y})}$$

Where the a_j's are the constants tabulated and presented in Shapiro and Wilk (42). The a_j's are independent of P. Malkovich, from his investigation, concluded that the W* statistic is invariant, with one as an upper bound, and that it reduces to W when the number of variates is one. In testing hypotheses, the assumption of multivariate normality is rejected for small values of W*. It is noted here that the critical values reported by Malkovich (31) appear to have been computed using

$$WP = \frac{\sum(a_j UU_j)^2}{(Y_m-\bar{Y})A^{-1}(Y_m-\bar{Y})}$$

instead of the form previously discussed. This result was discovered upon attempting to verify the computer routine used to calculate the W* statistic. Thus, if the correct formula for W* is used, it will always result in values greater than the values calculated using the form used by Malkovich to calculate the critical values. This would result in accepting the hypothesis of multivariate normality too often. As a result, this study uses estimates of critical values for W* (computed using the correct formulation) by Young (52).

Skewness (B1,P) and Kurtosis (B2,P) Statistics

The skewness and kurtosis statistics used in this study are those presented by Mardia (32,33). Only a brief discussion of these statistics is given here. Mardia presented the following forms for these statistics:

$$B1,P = E\{(X-\mu)'\textstyle\sum^{-1}(Y-\mu)\}^3$$

$$b1,P = \frac{1}{n^2}\sum_{i,j=1}^{n}\{X_i-\overline{X})'S^{-1}(X_j-\overline{X})\}^3$$

and

$$B2,P = E\{(X-\mu)'\textstyle\sum^{-1}(X-\mu)\}^2$$

$$b2,P = \frac{1}{n}\sum_{i=1}^{n}\{(X_i-\overline{X})'S^{-1}(X_i-\overline{X})\}^2$$

where b1,P and b2,P are the sample statistics for B1,P and B2,P, respectively. It is noted here that Mardia uses

$$S = \frac{1}{n}\sum_{i=1}^{n}(X_i-\overline{X})(X_i-\overline{X})'$$

as the sample covariance-variance matrix. However, this research uses the unbiased form given by

$$S' = \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \overline{X})(X_i - X)'$$

to calculate critical values and powers for this research. This form is selected as a matter of convenience since available computer programs for calculating the sample covariance-variance matrix are written using S'.

CHAPTER III

METHODOLOGY

Introduction

As stated previously, one of the principle objectives of this research is to develop power curve relationships that could be used by the decision maker to validate the assumption of multivariate normality for data that has already been collected or to better design multivariate experiments with regards to adequate sample size. However, in order to develop these power curve relationships it is necessary to have some knowledge of how the statistic to be used in the test behaves under the null hypothesis. Specifically, it is necessary to know the percentage points of a given statistic in order to calculate the powers of the test of the distribution at a particular α-level. But the distributions of multivariate statistics are complex and knowledge of them is limited. Thus, it is necessary to employ Monte Carlo techniques in determining necessary critical values as well as in the computation of powers. The general procedures for deriving critical values and powers along with necessary computer programs are discussed in the following sections.

Procedures for Computing Critical Values

Where necessary, critical values are computed as follows:

1. Take random samples from multivariate normal

distribution.

2. Compute the test statistic for each sample taken.

3. Rank order statistics.

4. For one-tailed tests, choose the values below which (above which, if the rejection region is in the upper tail as with B1,P) the desired percentage of test statistics fall. In the case of a two-tailed test, the critical region is divided in half with lower and upper critical values determined in the manner described for the one-tailed test. The resulting values are estimates for the $1-\alpha$ percentage points for the distribution of the test statistic used in (2).

Procedure for Computing Powers

Powers for all statistics are computed as follows:

1. Take random samples from non-normal multivariate distribution.

2. Compute test statistic for each sample.

3. Compare statistics to appropriate critical values for test statistic.

4. Total the number of times the test statistics cause the hypothesis that the sample came from a multivariate normal distribution to be rejected.

5. Divide total times rejected by number of samples taken. The result is an estimate of the power of the test against the non-normal multivariate distribution.

Alternative Distributions and Parameter Values

In order to develop power curve relationships, it is necessary to specify the alternative distribution. Since knowledge of the joint density of multivariate distributions is limited, this research focuses on marginal distributions in order to characterize multivariate distributions. In addition, the invariance of B1,P and B2,P shown by Mardia (33) and the invariance of W* shown by Malkovich (31) make it convenient and practical to use independent variates. Thus, alternative multivariate distributions for this study are formed as $\underset{\sim}{Y} = (Y_1,\ldots,Y_p)'$ where the Y_i for i=1,...,P are independently distributed and taken from the following univariate distributions with parameters fixed as indicated in Table 3.1.

These distributions and parameter values were selected as being representative of marginal distributions likely to be encountered by OTEA. When the marginal distributions of the alternative multivariate distribution are the same, that is, the Y_i are all from the same distribution, the resulting multivariate matrix ($\underset{\sim}{Y}$) will be referred to as a pure matrix. All other matrices will be referred to as mixed matrices.

Table 3.1. List of Univariate Distributions and Their Parameter Values

DISTRIBUTION	PROBABILITY DENSITY FUNCTION	PARAMETERS
1. Uniform	$1/b-a$	$b=1,\ a=0$
2. Beta	$\Gamma(P+Q)/[\Gamma(P)T'(Q)]X^{P-1}(1-X)^{Q-1}$	$P = 5.5,\ Q = 1.2$
3. Exponential	$\lambda e^{-\lambda x}$	$\lambda = 1.0$
4. Binomial	$\binom{N}{X} p^{x} q^{N-x}$	$N = 40,\ p = .65$ $q = .35$

Computation of Estimates of Power

A computer program composed of a main program and subroutine, calculates estimates of powers for the W* statistics against the alternative distributions discussed previously. The program allows the user to build the sample multivariate non-normal matrix with observations taken on any combination of the univariate distributions discussed in the previous section. The subroutine computes the W* statistics which are then compared to the critical values computed by Young (52). The powers are then computed following the general procedure previously discussed.

Powers for B1,P and B2,P are computed in the same manner as those for the W* statistic. However, since the necessary range of critical value estimates had not been tabulated, estimates for critical values for B1,P and B2,P are calculated as a part of this research. A computer program performs the power calculation. A listing of these programs is found in Appendix A. The results of the Monte Carlo studies are discussed in Chapter IV.

CHAPTER IV

MONTE CARLO STUDIES

Introduction

Monte Carlo runs for this study were conducted in three segments. The first segment consisted of deriving estimates of critical values for B1,P and B2,P along with determining an adequate number of samples for power estimation. The second segment consisted of deriving powers for B1,P, B2,P and W* against matrices composed of variates with like marginal distributions (pure matrices). The third segment consisted of deriving powers for each statistic against a range of mixed non-normal multivariate distributions.

Critical Values for B1,P and B2,P

For critical value estimation, the number of samples (500) used by Young (52) and Malkovich (31) are used. Following the format of the Young study, estimates of the critical values for B1,P and B2,P were computed for N=4,5,...,20, P=2,3,4,8 and α=.01,.02,.05,.10,.40,.90,.95, .98,.99.

It is noted again that the estimates found in these tables differ from those reported by Mardia (32) because this study uses the unbiased formula for computing the sample variance-covariance matrix, whereas Mardia used a biased formula. (See Chapter II for a detailed discussion of computational formulas.) Nevertheless, estimates of critical values found in this research were reconciled with those reported by Mardia, taking into account differences in computational methods, and it was found that the estimates were comparable. However, it is emphasized that in order to obtain meaningful results using these tables, the unbiased version of the sample variance-covariance matrix should be used. It is further noted that the estimates of the critical values for B1,P exhibit somewhat unusual behavior in that for P=2,3 and 4, the estimates begin small, increase and then decrease again as N increases for a given α-level. After carefully checking the computer program used to compute B1,P and comparing estimates with those of Mardia (Mardia does not report estimates for N less than 10), it appears that this seemingly abnormal behavior is characteristic of the B1,P statistic. Tabulated critical value estimates for B1,P and B2,P are given in Appendix B along with the critical value estimates for W* derived by Young.

Number of Samples for Power Estimation

A question which arose when following the procedure discussed in Chapter II was the appropriate number of samples to be taken in calculating estimates of powers. In order to keep computer costs within the budgeted amount, this research used the smallest number of samples possible that would still allow for reasonable accuracy. For power determination, sample powers were computed and plotted in an effort to determine the number of samples at which the estimates of the power stabilized. Based on this heuristic procedure, 350 samples were found to yield adequate estimates with minimum use of computer time. An example of the plots used to determine the number of samples to be taken is shown in Figure 4.1. It can be seen from this plot that power estimates tend to level off when the sample size is increased to 350. It was found that this number of samples yields powers which are accurate to $\pm$ .05.

Results of Power Study

The Monte Carlo power results for B1,P, B2,P and W* for the alternative distributions considered in Chapter III are presented in Appendices C-E. Power estimates of the test statistics are given at α = .01, .02, .05, .10, .50, .90, .95, .98, .99; for p = 2, 3, 4, 8; and n = 4, 5, 6, 7, 10, 12, 14, 16, 18, 20.

Appendix C contains tabulated estimated powers for B1,P, B2,P and W* against pure alternative distributions. Appendix D contains estimates of powers for B1,P, B2,P and W* against mixed alternative distributions.

The following mixed alternative distributions were considered.

UNIFORM	BETA	EXPONENTIAL	BINOMIAL	NORMAL	P
2		1			3
1		1		1	3
		1	1	1	3
1		1	1		3
2		2		1	3
		2		1	3
			2	1	3
	2			1	3
1	1	1	1		4

Computer costs prohibited consideration of more combinations. Appendix E contains plots of power versus alpha level for fixed P and N and plots of power versus N for fixed P and α. The following sections give

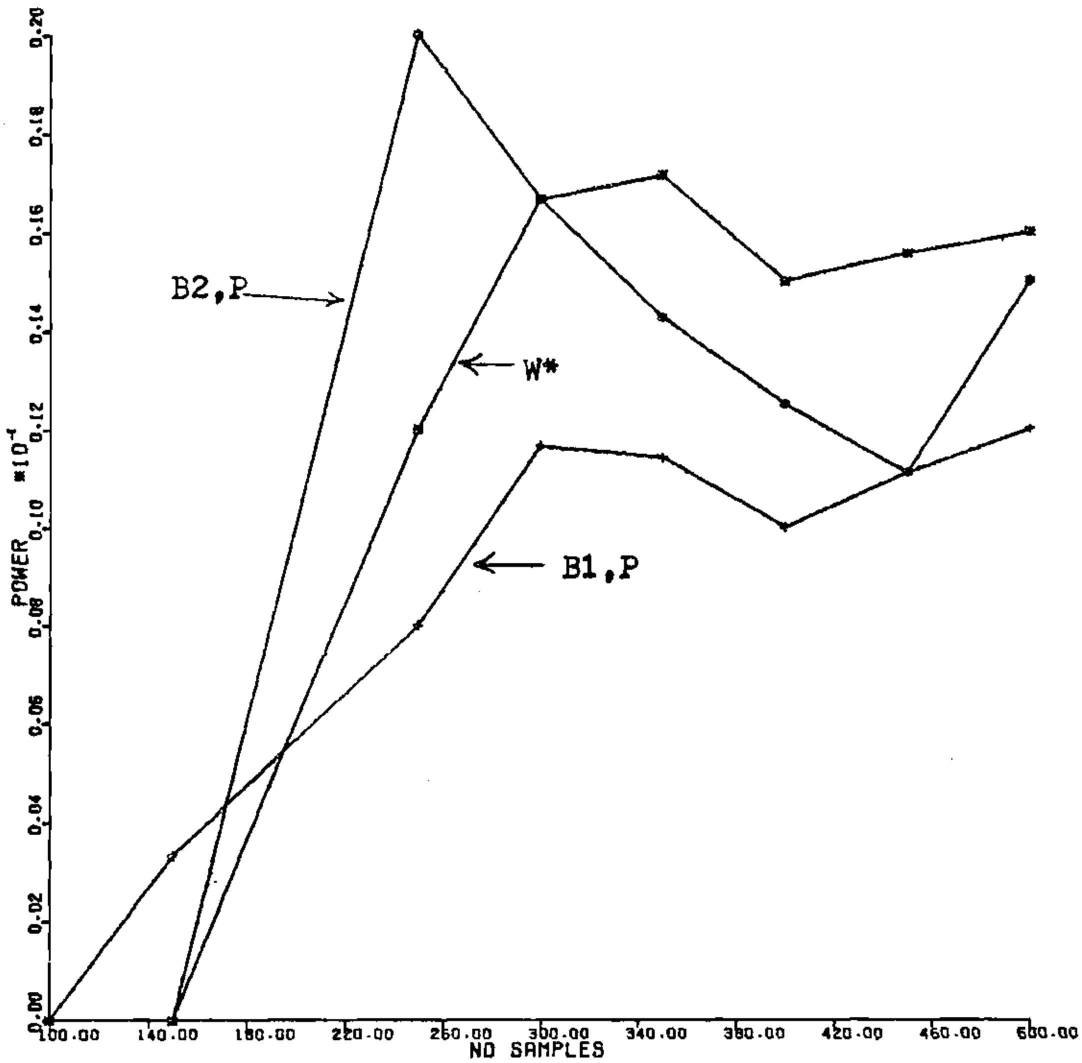

Figure 4.1. Sample Plot to Determine Stabilization of Powers, Alpha=.02

general remarks and comments summarizing comparisons made as a result of these Monte Carlo studies.

Powers Against Pure Alternative Distribution

In general, B1,P, B2,P and W* perform poorly against matrices composed of all uniform variates. This seems to be consistent with results reported by Malkovich (30,31) when the alternative distribution consisted of uniform variates. Of the three statistics used for this study, B2,P performs best against pure uniform matrices for N greater than 7. Figure 4.2 illustrates the dominance of B2,P over B1,P and W* against uniform variates. Figure 4.2 is a graph of power versus alpha for P=4 and N=20. The dominance of B2,P over B1,P and W* may also be seen in Figure 4.3, a graph of power versus N for P=4 and α=.10. From this figure it is seen that as N increases beyond 7, B2,P yields higher powers. Although, for N less than 7, W* seems to yield slightly higher powers, due to the $\pm$ .05 tolerance in estimated powers there is no significant difference in the performance of the statistics in this range. (See Figure 4.4.)

Against pure beta variates as with pure uniform variates there appears to be no significant difference between performance for small values of N and P (see Figures 4.5 and 4.6). However, as P and N increase, B1,P seems to dominate both B2,P and W*. It is emphasized at this point that although specific cases are cited, they are representative of the general behavior of power for the statistics under consideration.

For pure exponential variates B1,P performs at least as well as

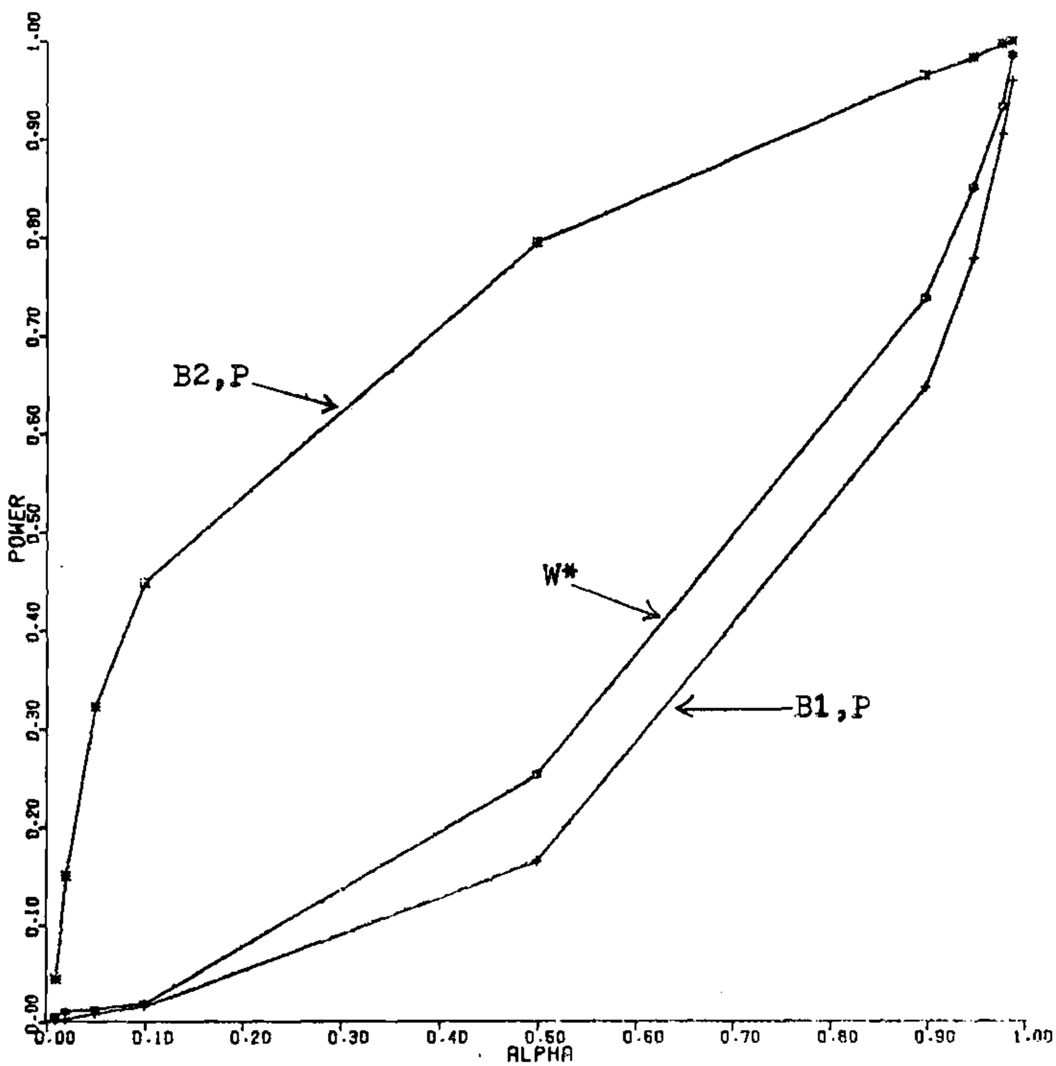

Figure 4.2. Plot of B1,P, B2,P and W* Against Uniform Variates, P=4, N=20

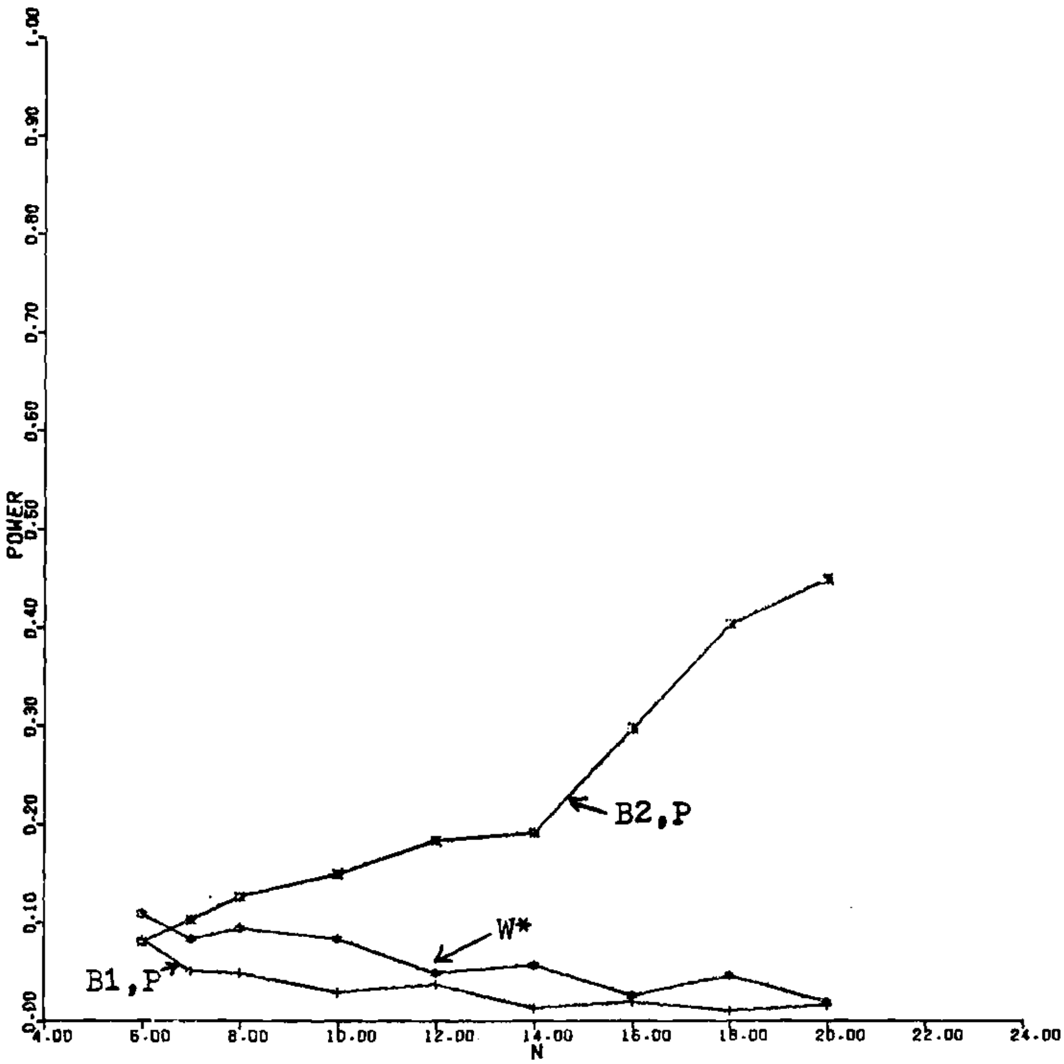

Figure 4.3. Plot of B1,P, B2,P and W* Against Uniform Variates, P=4, Alpha=.10

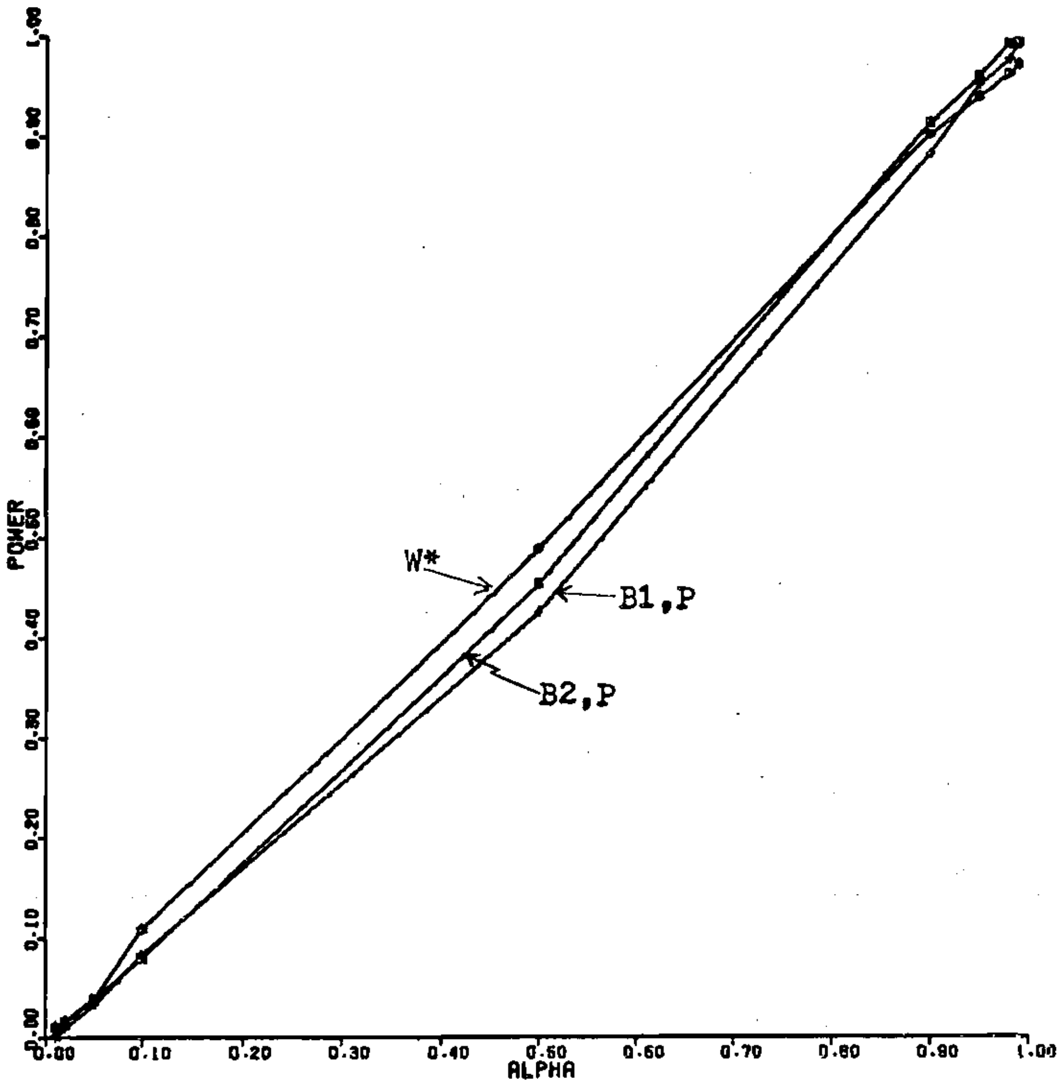

Figure 4.4. Plot of B1,P, B2,P and W* Against Uniform Variates, P=4, N=6

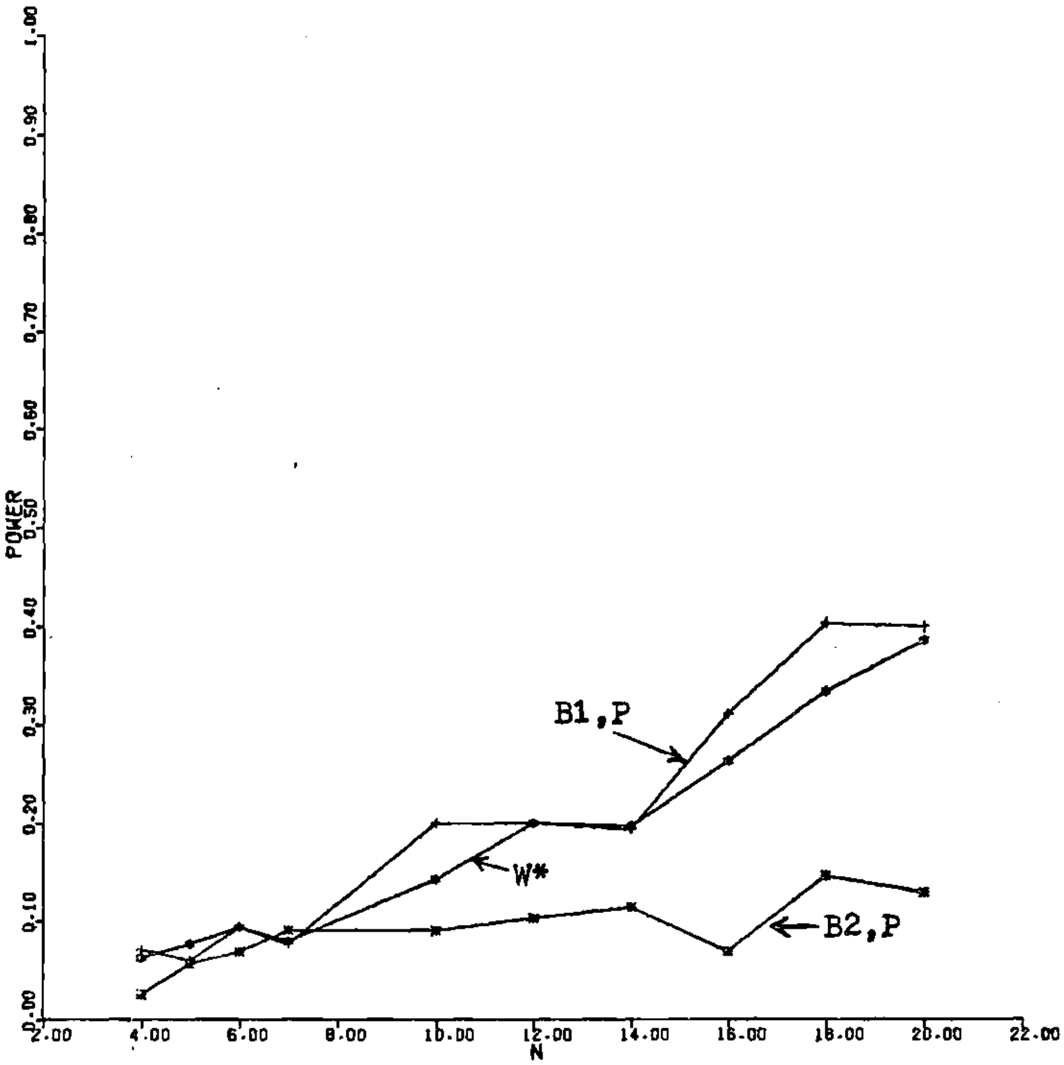

Figure 4.5. Plot of B1,P, B2,P and W* Against Beta Variates, P=2, Alpha=.05

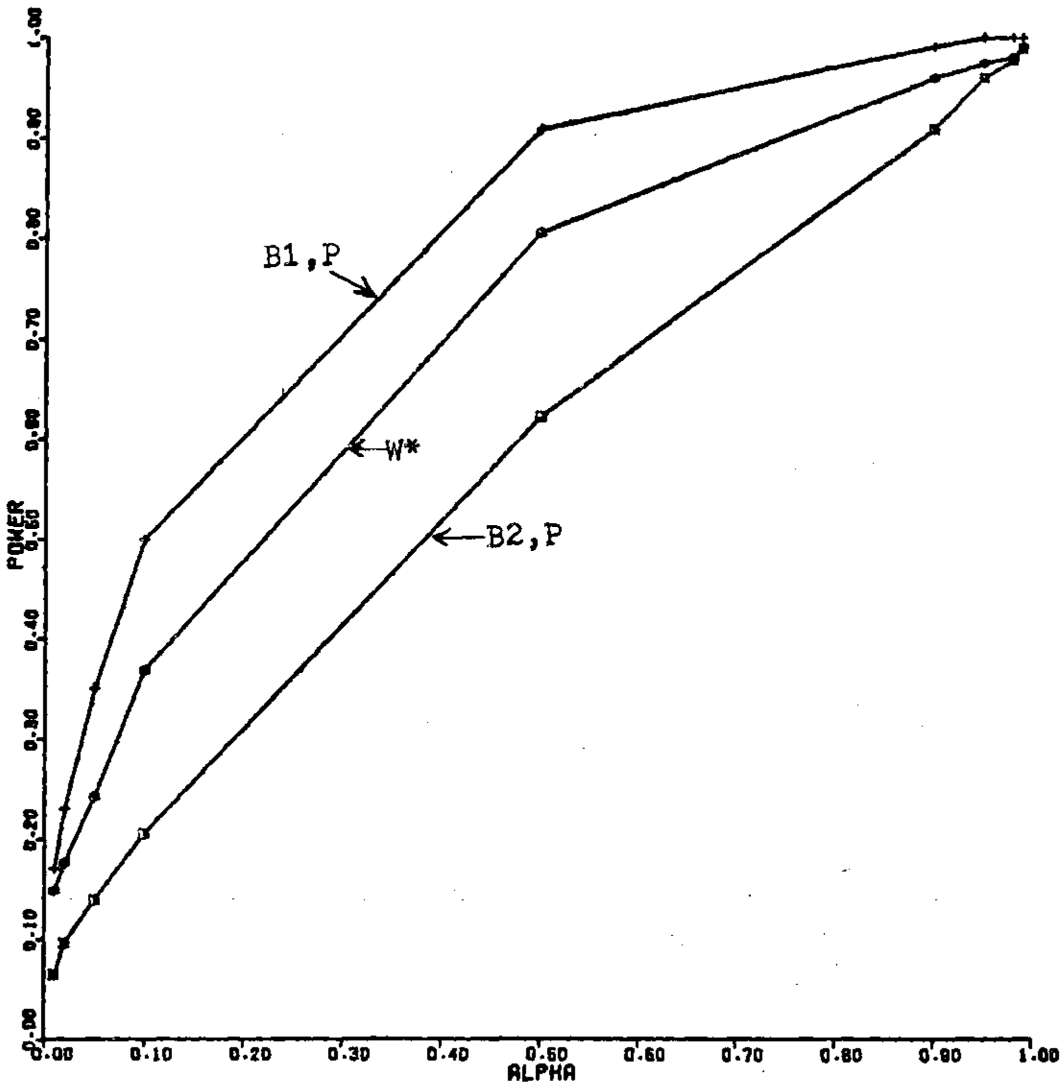

Figure 4.6. Plot of B1,P, B2,P and W* Against Beta Variates, P=4, N=20

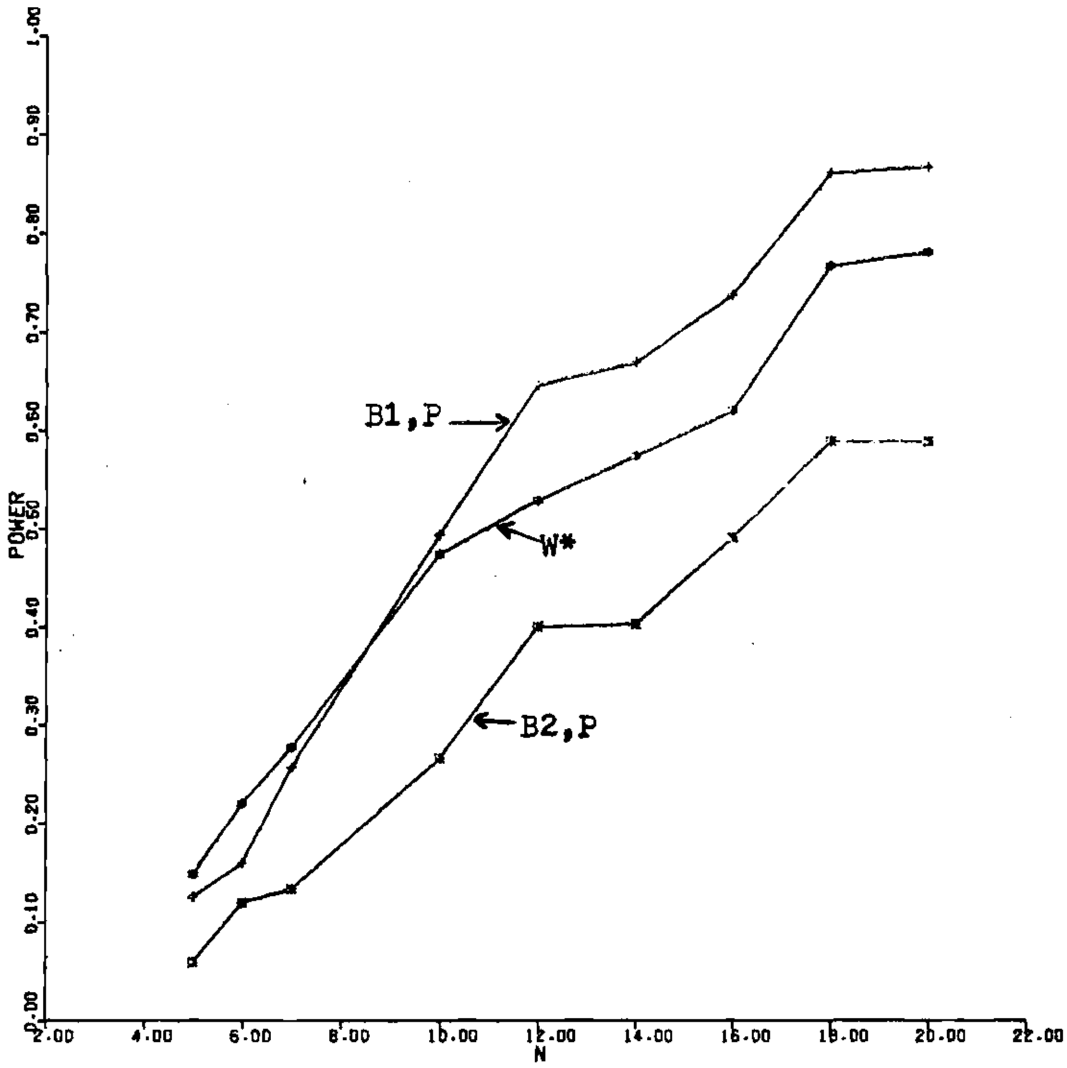

Figure 4.7. Plot of B1,P, B2,P and W* Against Exponential Variates, P=3, Alpha=.10

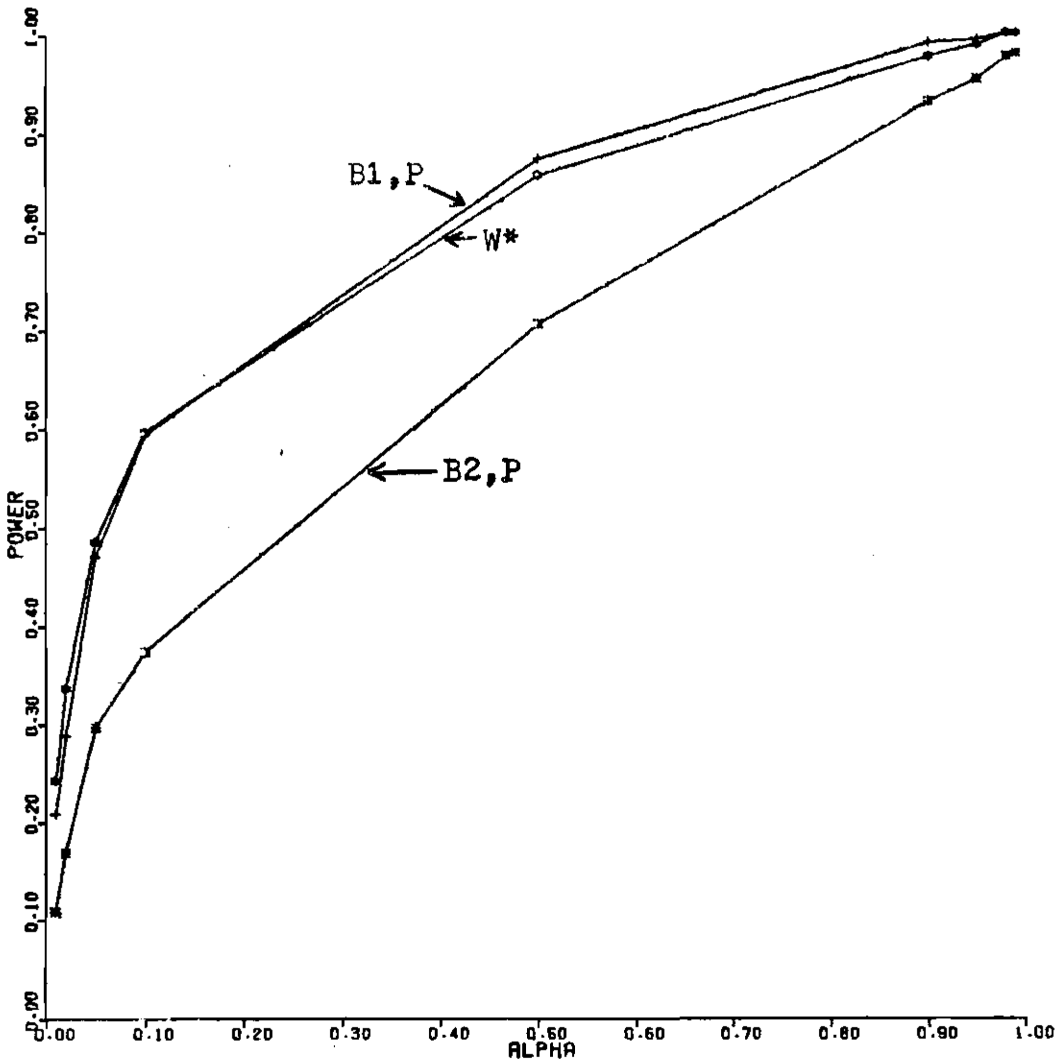

Figure 4.8. Plot of B1,P, B2,P and W* Against Exponential Variates, P=2, N=12

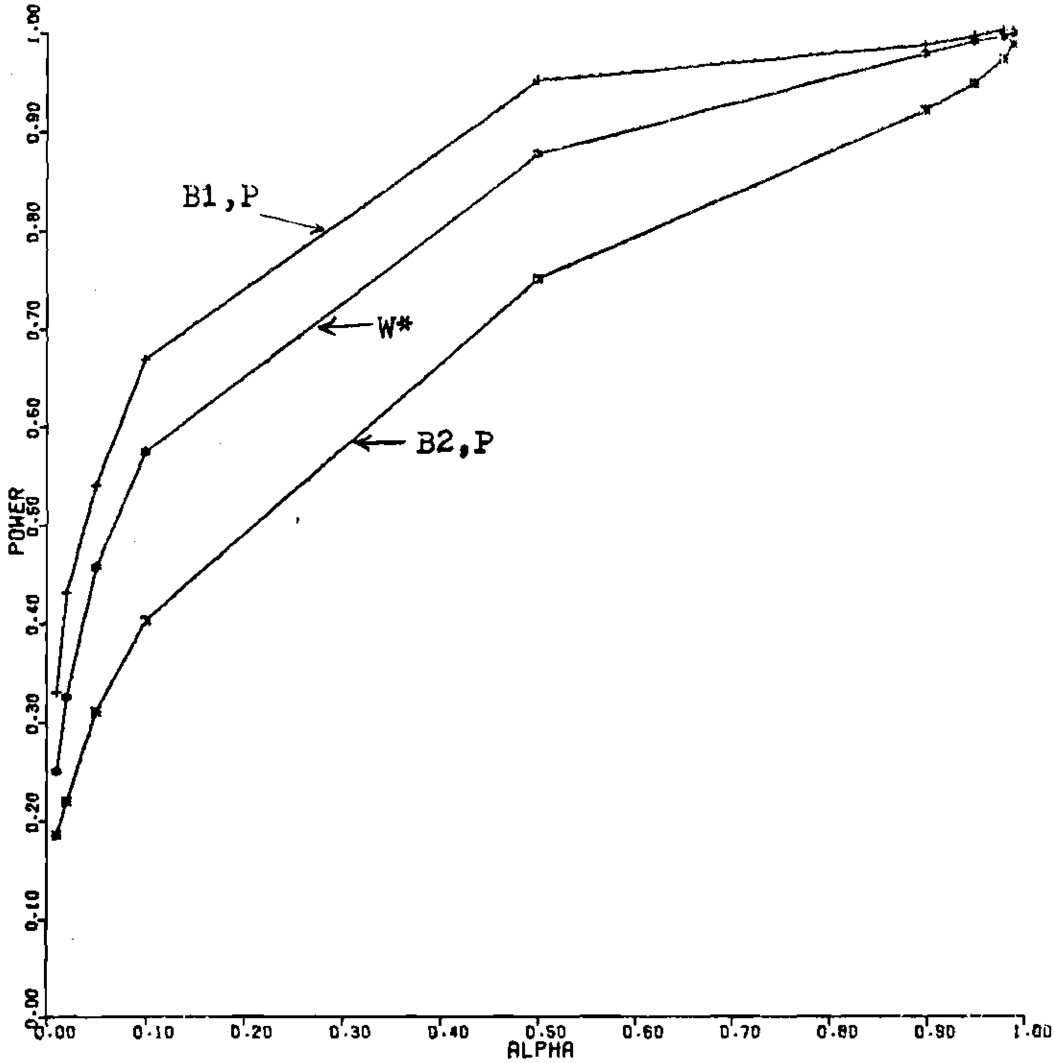

Figure 4.9. Plot of B1,P, B2,P and W* Against Exponential Variates, P=3, N=14

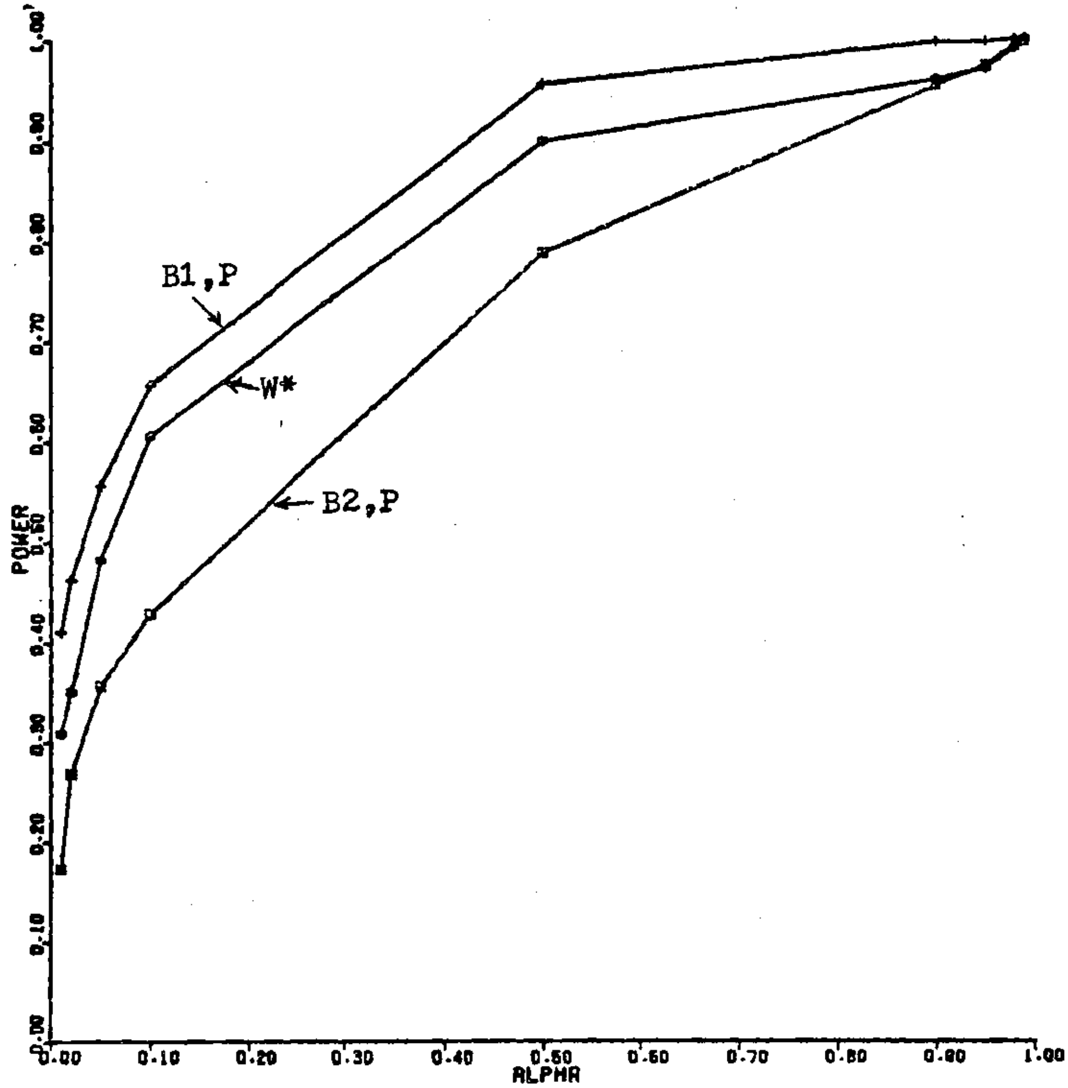

Figure 4.10. Plot of B1,P, B2,P and W* Against Exponential Variates, P=4, N=14

B2,P and W* for N greater than 7 and alpha greater than .10 (see Figures 4.7 and 4.8). Figures 4.9 and 4.10 indicate that for higher values of P (i.e. P greater than 2) B1,P yields uniformly greater powers than B2,P and W*.

All statistics considered perform poorly against binomial variates, with no increase in power with increased sample size (see Figure 4.11). Results against pure variates are summarized in Table 4.1.

Powers Against Mixed Alternative Distributions

Examining the estimates of the powers for the mixed alternative distributions considered in this study, it appears that the W* statistic performs at least as well as B1,P and B2,P for all cases considered except when the mixed alternative distribution is composed of uniform and normal or binomial and normal variates.

In general, it seems that all statistics considered perform best when at least one variate is either exponential or beta. When mixed alternative distributions are composed of combinations of uniform, normal and binomial variates, there seems to be a significant drop in power estimates. Thus, based on these results, the following ranking of variates in terms making detection of multivariate non-normality seems appropriate (see Table 4.2).

In addition, it appears that if a mixed alternative distribution contains P variates then the powers against such a distribution are comparable to powers against an alternative distribution composed of P-1 of the most simple marginal to detect in the mixed alternative distribution. These results suggest the following heuristic procedure for powers against mixed alternative multivariate distributions:

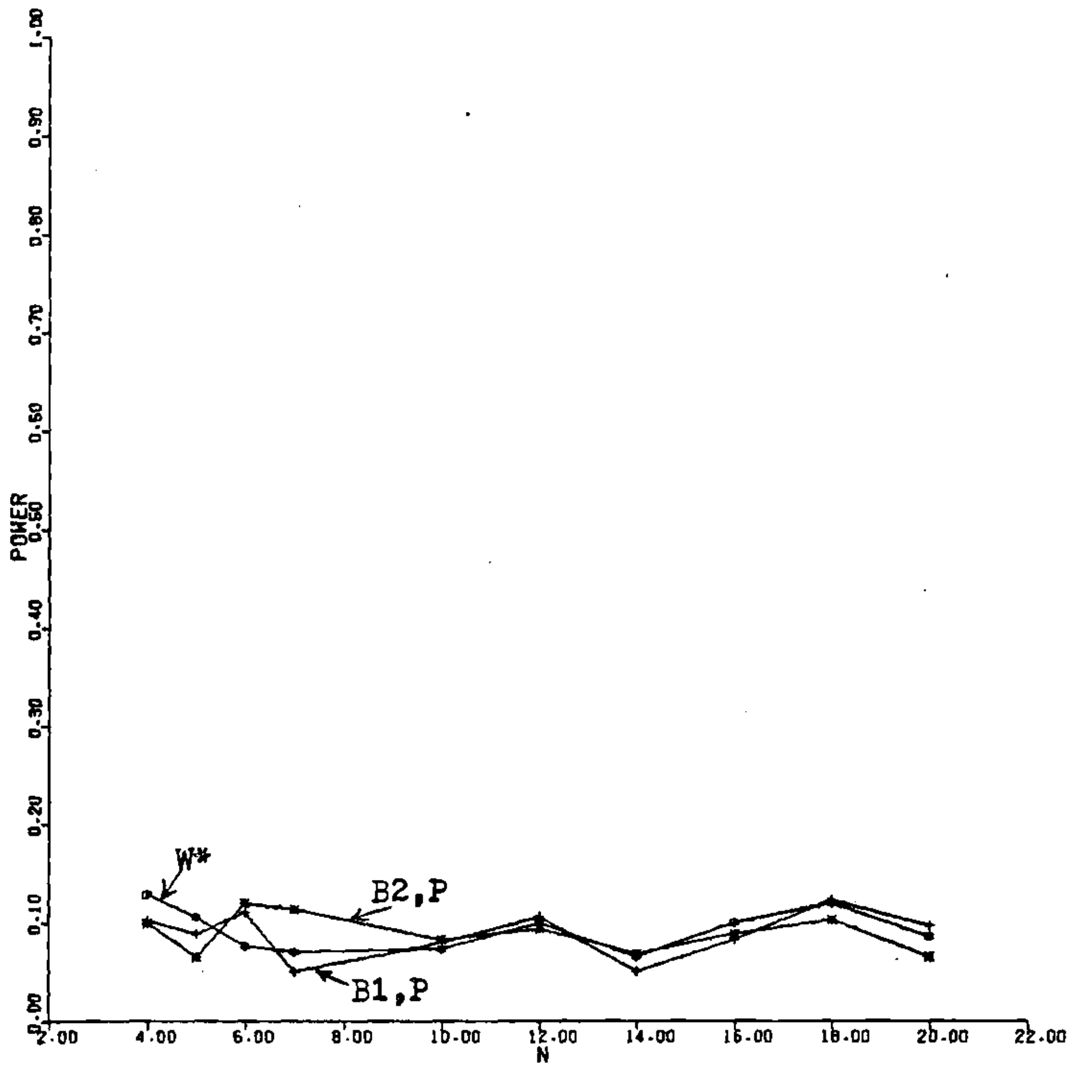

Figure 4.11. Plot of B1,P, B2,P and W* Against Binomial Variates, P=2, Alpha=.10

Table 4.1. Best Performing Statistics Against Pure Alternative Distributions

Range of N	Uniform	Beta	Exponential	Binomial
N<7	Indifferent	Indifferent	Indifferent	Indifferent
N>7	B2,P	B1,P	B1,P	Indifferent

Table 4.2. Rank Ordering of Variates in Terms of Facilitation of Detection of Multivariate Non-normality

Distributions

Exponential		Beta		Binomial/Uniform
EASIEST TO DETECT	→		→	MOST DIFFICULT TO DETECT

1. Determine marginal distributions based on multivariate data. This can be done using conventional methods such as plotting histograms of individual variates.
2. Select easiest detection variate according to a scheme such as Table 4.2.
3. Use power tables for pure variates of the type selected in (2) to approximate powers against the mixed alternative distribution. This is done by entering the pure tables at P=I, where I is the number of easiest detection variates in the sample.

This procedure is practical in that it would make it unnecessary to tabulate power estimates for the many possible combinations of variates. Thus, the user could rely on pure tables for power estimates for both pure and mixed cases. A demonstration of the results and this procedure is discussed in Chapter V.

CHAPTER V

AN APPLICATION TO OPERATIONAL TESTING

Introduction

This chapter will give a demonstration of the power curve relationships reported in Chapter IV to an operational testing problem. The basis for this demonstration will be data from Lightweight Company Mortar System, XM224E3, Operational Test II (LWCMS OT II), Report: FTR-OT-027 (48).

Background

The LWCMS OT II was designed to compare the 81mm mortar system to the LWCMS. Although the test consisted of five phases (pretest and transition training, pilot test, field training exercise, controlled line firing exercise and parachute delivery demonstration), this application focuses on the controlled line firing exercise. Specifically, this demonstration assumes that the commander, U.S. Army Operational Test and Evaluation Agency (OTEA), was concerned with the overall effectiveness of the LWCMS. Thus, the following MOE's are assumed to be approved as relevant MOE's:

1. Time in seconds for the forward observer (FO) to prepare and call for fire.
2. Time for the Fire Direction Center (FDC) to prepare plots and transmit information to the

guns.

3. Number of rounds needed to complete adjustment prior to firing all guns for effect.

This demonstration addresses the question of whether the above MOE's may be assumed to be multivariate normal as well as what sample size is sufficient for testing the assumption of multivariate normality.

Example I

The data in Table 5.1 are from OT II and serve as the basis for the first application.

Based on histograms of the MOE provided in the OT II report, it seems that MOE's #1 and #2 could have exponential marginal distributions, while #3 seems to have a uniform marginal distribution. Following the heuristic procedure outlined in Chapter IV, since exponential variates provide easier detection than uniform variates, the power tables against pure exponential variates is used. Furthermore, since P for the mixed case is 3, the power tables against pure exponential variates is entered with P=2 (I for mixed case =1), N=7 and a hypothetical α-level of .10. From the tables in Appendix C it is found that estimates of powers for B1,P, B2,P and W* are .229, .186 and .251, respectively. These powers are small, however, powers for B1,P, B2,P and W* on the order of .874, .486 and .837 could be achieved by increasing the sample size to 20.

From the data, the following sample variance-covariance matrix and estimates of B1,P$(\widehat{B1,P})$, B2,P$(\widehat{B2,P})$ and W*$(\widehat{W^*})$ were computed.

Table 5.1. Data for Example I

	Mission Number						
MOE	1007	10011	10015	10029	40005	40015	40105
1. Time for FO TO CALL(SEC)	110	405	115	105	75	53	72
2. Time for Plots Info to Guns (SEC)	144	187	99	17	155	137	83
3. No. of Rounds to Adjust	7	4	5	4	4	2	5

$$S' = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 14850.62 & 2630.45 & 1.71 \\ 2 & 2630.45 & 3841.95 & -22.28 \\ 3 & 1.71 & -22.28 & 2.28 \end{array}$$

$$\widehat{B1,P} = 5.199 \qquad \widehat{B2,P} = 9.943 \qquad \hat{W}^* = .554$$

A comparison of the test statistics to their respective ($1 - \alpha$) percentage for P=3, N=7 and α=.10 (see Appendix B) indicates that the assumption of multivariate normality should be rejected. Since, the power estimates were small, it perhaps indicates that the marginals were not as assumed or that detection of non-normality was a fortunate occurrence.

Example II

Another example of how the results of Chapter IV may be applied is given here. The data listed in Table 5.2 was also obtained from the OT-II report.

These data are average squad performance times for the indicated MOE's using the LWCMS. Since the LWCMS was compared to the 81mm mortar in the OT-II report, it is assumed, as before, that the commander would be interested in the validation of the assumption of multivariate normality to determine whether the mean vectors for the two systems could be compared using multivariate statistical techniques. Since the MOE's are times and nonnegative, the alternative distribution is assumed to be composed of exponential variates. Thus, from the tables in Appendix C estimates of the powers are .16, .12 and .22 for B1,P, B2,P and W* for α=.10, N=6 and P=3.

Table 5.2. Data for Example II

	Squad/Company					
MOE	1/A	2/A	3/A	1/B	2/B	3/B
1. Mounting (SEC) the Mortar	58.2	63.0	53.0	62.9	58.0	64.5
2. Small def/dec (SEC)	24.4	25.5	24.1	21.8	22.5	24.0
3. Large def/elev. (SEC) charge	25.7	25.4	20.7	26.4	21.8	22.2

The sample variance-covariance matrix and estimates of B1,P, B2,P and W* are given below:

$$S' = \begin{array}{c|ccc} & 1 & 2 & 3 \\ 1 & 18.774 & -.83 & 5.475 \\ 2 & -.083 & 1.805 & -.008 \\ 3 & 5.475 & -.008 & 5.808 \end{array}$$

$$\widehat{B1,P} = 1.790 \qquad \widehat{B2,P} = 6.675 \qquad \widehat{W^*} = .893$$

Comparing the test statistics to their respective $(1-\alpha)$ percentage points for P=3, N=6 and α=.10 found in the tables in Appendix B, it appears that the assumption of multivariate normality cannot be rejected for any of the test statistics computed. However, the power estimates seem small for this sample size. Thus it would be recommended that other squads be tested to provide a larger sample size in order to attain greater confidence in either failing to reject the assumption of multivariate normality or accepting, if these powers are not satisfactory.

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

This research has addressed the problem of the validation of the assumption of multivariate normality and sample size determination. Estimates of critical values for B1,P and B2,P have been derived in order to employ Monte Carlo techniques to estimate powers. Monte Carlo power studies were conducted and power estimates derived for three statistics (B1,P, B2,P and W*) for P=2,3,4,8, N=4,...,20 and α=.01,...,.99.

From the power studies presented in this research it was seen that no statistic considered dominates the others in all cases. It was further observed that alternative distributions composed of certain types of variates (i.e. exponential variates) are easier to detect. Thus, it is possible to rank order variates in terms of their ease of detection. In the case of mixed marginal distributions, a heuristic procedure was developed which enables the user to utilize tables of power estimates for pure variates. This procedure was based on the observation that certain variates in the mixed case tend to characterize the multivariate distribution.

It is possible to study multivariate power relationships by examining the marginal distributions of the alternative multivariate distribution. Examination of the marginal distributions facilitates the study of power relationships, since knowledge of the joint density of multivariate distributions is limited.

Limitations of the Research

This research has been limited by high computer costs associated with empirically determining estimates of powers. These costs led to a trade-off between high accuracy in power estimation and a wide ranging survey of interesting cases. In this study, high accuracy was sacrificed in order to gain a broader understanding of power curve relationships for the statistics under consideration.

This research was also limited by the difficulty in specifying alternative distributions. As seen from the examples given in Chapter V, accurate specification of the marginals which form the alternative distribution is essential in order to identify the proper power tables.

Recommendations

Several recommendations for future research are made as a result of this study. The first recommendation is that a study of robustness be conducted for non-normal alternative multivariate distributions (such as ones composed of uniform marginals) to determine the effect of not being able to detect these types of non-normal alternative distributions. This recommendation is made because from this study it was found that distributions composed of certain marginals are virtually impossible to detect. Thus it seems that robustness studies would be valuable in determining those cases where inability to detect multivariate normality does not interfere with the meaningful use of other multivariate techniques that depend on the normality assumption. Perhaps loss functions could be developed for cases where detection of multivariate normality is difficult.

In addition, investigations should be conducted to determine what procedures should be taken once it is determined that the hypothesis of multivariate normality cannot be rejected. There seem to be three alternatives when the hypothesis of normality is rejected. The first would be to go ahead and use multivariate techniques, assuming that the data is robust to the assumption of multivariate normality. Second, transformations of the data could be made using this research as a basis and following procedures similar to those discussed by Box and Cox (11). Finally, it may be deemed necessary to use some other form of statistical analysis in lieu of multivariate methods. The objective of studies of this type would be to determine under what conditions each alternative is preferred to the others.

Another recommendation is that more accurate studies be conducted using mixed alternatives in an effort to better determine which type marginal distributions characterize or dominate the mixed distribution. The goal in this case would be to develop more precise guidance in those cases where the alternative distribution is of a mixed nature.

In addition, it was observed that for certain alternative distributions (i.e. pure binomial marginals) power seems to remain constant as sample size increases. This observation is extremely important in those cases where the marginal cost of additional samples is high. Thus, further study should be directed toward determining the sensitivity of powers to design parameters such as sample size.

Finally, it is recommended that an investigation be conducted to determine whether it is feasible to develop a validation procedure based primarily on an analysis of the nature of marginal distributions. This recommendation is made in view of the observation that there seems to be differences in the ability to detect multivariate non-normality depending on the types of marginals encountered. Thus, a procedure based on the analysis of marginals could reduce the need for extensive tables of power estimates and could be used in conjunction with existing tests for multivariate normality.

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APPENDICES

APPENDIX A

This appendix contains a complete FORTRAN IV listing of the computer programs developed by the author to implement the methodology for the power studies described in Chapter III. The first program and subroutine calculate estimates of powers for W*. The second program calculates critical value estimates for B1,P and B2,P, while the third program calculates power estimates for these statistics.

```
 PROGRAM THESIS(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE15)
 DIMENSION R(10),X(10,30),D1(5,30),D2(5,30),D3(5,30),
+D4(5,30),D5(5,30),US(30),A(30),YBAR(10),DIFF(10,30),
+IPR(30),AMAT(10,10),AINVER(10,10),TEMPD(10,30),RMAX(30),
+PMAT(10,10),QBAR(10),SUMSQ(10,10),ASK(11,19),AK2(20,10),
+AK3(20,10),AK4(20,10),POW(9),POWER(9)
 DATA ((ASK(IG,JG),JG=1,19),IG=1,8)/.7071,.7071,.6872,
+.6646,.6431,.6233,.6052,.5885,.5739,.5601,.5475,.5359,
+.5251,.5150,.5056,.4968,.4886,.4808,.4734,
+1.0,.0,.1677,.2413,.25066,.3031,.3164,.3244,.3291,.3315,
+.3325,.3325,.3318,.3306,.3290,.3273,.3253,.3232,.3211,
+.0,1.0,1.0,.0,.0875,.1401,.1743,.1976,.2141,.2260,.2347,
+.2412,.2460,.2495,.2521,.2540,.2553,.2561,.2565,
+.0,.0,.0,1.0,1.0,.0,.0561,.0947,.1224,.1429,.15866,.1707,
+.1802,.1878,.1939,.1988,.2027,.2059,.2085,
+.0,.0,.0,.0,.0,1.0,1.0,.0,.0399,.0695,.0922,.1099,.1240,
+.1353,.1447,.1524,.1587,.1641,.1686,
+.0,.0,.0,.0,.0,.0,.0,1.0,1.0,.0,.0303,.0539,.0727,.0880,
+.1005,.1109,.1197,.1271,.1334,
+.0,.0,.0,.0,.0,.0,.0,.0,.0,1.0,1.0,.0,.0240,.0433,.0593,
+.0725,.0837,.0932,.1013,
+.0,.0,.0,.0,.0,.0,.0,.0,.0,.0,.0,1.0,1.0,.0,.0196,.0359,
+.0496,.0612,.0711/
 DATA((ASK(IB,JB),JB=1,19),IB=9,11)/.0,.0,.0,.0,.0,
+.0,.0,.0,.0,.0,.0,.0,.0,1.0,1.0,.0,.0163,.0303,.0422,
+.0,.0,.0,.0,.0,.0,.0,.0,.0,.0,.0,.0,.0,.0,.0,1.0,1.0,.0,.0140,
+.0,.0,.0,.0,.0,.0,.0,.0,.0,.0,.0,.0,.0,.0,.0,.0,.0,1.0,1.0/
 DATA((AK2(I2,J2),J2=1,9),I2=4,7)/.6315,.6341,.6469,.6615,
+.7955,.9495,.9702,.9919,.9952,.6056,.6233,.6539,.6802,
+.8507,.9562,.9643,.9735,.9806,.5938,.6310,.6829,.7222,.8770,
+.9549,.9676,.9797,.9876,.6298,.6637,.7071,.7575,.8878,
+.9564,.9545,.9790,.9814/
 DATA ((AK2(II2,JJ2),JJ2=1,9),II2=10,10)/.6696,.6939,.7717,.8062,
+.9052,.9535,.9595,.9764,.9821/
 DATA ((AK2(III2,JJJ2),JJJ2=1,9),III2=12,12)/.7422,.7765,.8110,
+.8443,.9263,.9664,.9726,.9821,.9845/
 DATA ((AK2(IL2,JL2),JL2=1,9),IL2=14,14)/.7692,.7811,.8259,
+.8564,.9323,.9685,.9740,.9780,.9813/
 DATA ((AK2(IZ2,JZ2),JZ2=1,9),IZ2=16,16)/.7924,.8125,.8511,
+.8770,.9392,.9715,.9767,.9824,.9835/
 DATA ((AK2(IB2,JB2),JB2=1,9),IB2=18,18)/.8066,.8292,.8610,
+.8689,.9464,.9734,.9785,.9833,.9850/
 DATA ((AK2(IC2,JC2),JC2=1,9),IC2=20,20)/.8410,.8535,.8755,
+.8951,.9505,.9750,.9797,.9834,.9866/
 DATA((AK3(I3,J3),J3=1,9),I3=5,7)/.5543,.5557,.5631,.5737,
+.6668,.8654,.9225,.9524,.9751,.5397,.5577,.5779,.6036,
+.7668,.9256,.9470,.9718,.9818,.5370,.5607,.6148,.6516,
+.8091,.9255,.9417,.9586,.9695/
 DATA ((AK3(II3,JJ3),JJ3=1,9),II3=10,10)/.6349,.6678,.7194,
+.7598,.8723,.9408,.9553,.9644,.9767/
 DATA ((AK3(III3,JJJ3),JJJ3=1,9),III3=12,12)/.6618,.7207,.7592,
+.7837,.8871,.9447,.9575,.9689,.9796/
 DATA ((AK3(IL3,JL3),JL3=1,9),IL3=14,14)/.7061,.7300,.7751,
+.8101,.8959,.9519,.9605,.9684,.9731/
 DATA ((AK3(IZ3,JZ3),JZ3=1,9),IZ3=16,16)/.7467,.7708,.8001,
+.8342,.9099,.9547,.9629,.9716,.9769/
 DATA ((AK3(IB3,JB3),JB3=1,9),IB3=18,18)/.7308,.7731,.8237,.8539,
+.9210,.9615,.9678,.9735,.9762/
 DATA ((AK3(IC3,JC3),JC3=1,9),IC3=20,20)/.7720,.7954,.8299,
+.8593,.9269,.9650,.9695,.9752,.9793/
 DATA ((AK4(I4,J4),J4=1,9),I4=6,8)/.4975,.4984,.5013,.5088,
+.5776,.7932,.8859,.9297,.9511,.4864,.4878,.5095,.5417,
+.6612,.8422,.9176,.9563,.9723,.4875,.5206,.5574,.5866,
+.7301,.8593,.9200,.9504,.9659/
```

```
      DATA ((AK4(II4,JJ4),JJ4=1,9),II4=10,10)/.5317,.5887,.6544,
     +.6891,.8[illegible],.9[illegible]9,.9346,.96[illegible]8,.9698/
      DATA ((AK4(III4,JJJ4),JJJ4=1,9),III4=12,12)/.5977,.6363,.6776,
     +.7255,.83[illegible],.9[illegible]91,.9282,.9567,.9626/
      DATA ((AK4(IL4,JL4),JL4=1,9),IL4=14,14)/.6693,.6919,.7414,
     +.7712,.8569,.9211,.9335,.9623,.9756/
      DATA ((AK4(IZ4,JZ4),JZ4=1,9),IZ4=16,16)/.6812,.7262,.7659,
     +.7947,.87[illegible]9,.9412,.9533,.9643,.9672/
      DATA ((AK4(IB4,JB4),JB4=1,9),IB4=18,18)/.7335,.7507,.7925,
     +.8127,.8851,.9411,.9515,.9649,.9706/
      DATA ((AK4(IC4,JC4),JC4=1,9),IC4=20,20)/.7667,.7776,.8012,
     +.8281,.9[illegible]12,.9476,.9569,.9645,.9725/
      WRITE(15,941)
941   FORMAT(17X,"EMPIRICALLY DERIVED POWERS FOR THE W* STATISTIC",
     +//,34X,"ALPHA LEVELS")
      WRITE(15,942)
942   FORMAT(8X,".01",5X,".02",5X,".05",5X,".10",5X,
     +".50",5X,".90",5X,".95",5X,".98",5X,".99")
      WRITE(15,943)
943   FORMAT(//,1X,"N",37X,"P=3")
80    READ(5,*)NP,N,N1,N2,N3,N4,N5,ISEED,NTOT,P,Q,XM,NB,PB
      PRINT(6,*)NP,N,N1,N2,N3,N4,N5,ISEED,NTOT,P,Q,XM,NB,PB
      KK=NP
      NN=N
      DO 12 JIM=1,N
      IF(ASK(JIM,N-1).EQ.1.0) GO TO 13
      A(JIM)=ASK(JIM,N-1)
12    CONTINUE
13    DO 14 JOE=JIM,N
      A(JOE)=-ASK((N-JOE+1),(N-1))
14    CONTINUE
      DO 15 I=1,9
      DO 20 J=1,100
      X(I,J)=0.0
      POW(I)=0.0
20    CONTINUE
15    CONTINUE
      DO 78 IKEEP=1,NTOT
      IF(N1.EQ.0)GO TO 70
      DO 81 IM=1,N1
      CALL GGUB(ISEED,N,R)
      DO 82 JM=1,N
      D1(IM,JM)=R(JM)
82    CONTINUE
81    CONTINUE
70    IF(N2.EQ.0) GO TO 71
      DO 83 ISW=1,N2
      CALL GGBTA(ISEED,P,Q,N,R)
      DO 84 JSW=1,N
      D2(ISW,JSW)=R(JSW)
84    CONTINUE
83    CONTINUE
71    IF(N3.EQ.0) GO TO 72
      DO 85 IYT=1,N3
      CALL GGEXP(ISEED,XM,N,R)
      DO 86 JYT=1,N
      D3(IYT,JYT)=R(JYT)
86    CONTINUE
85    CONTINUE
72    IF(N4.EQ.0) GO TO 73
      DO 87 IOP=1,N4
      DO 88 JOP=1,N
      D4(IOP,JOP)=GGBIN(ISEED,NB,PB)
88    CONTINUE
```

```
87     CONTINUE
73     IF(N5.EQ.0) GO TO 110
       DO 111 IMAR=1,N5
       CALL GGNOF(ISEED,N,R)
       DO 112 JMAR=1,N
       D5(IMAR,JMAR)=R(JMAR)
112    CONTINUE
111    CONTINUE
110     IF(N1.EQ.0) GO TO 74
       DO 21 IR=1,N1
       DO 22 IT=1,N
       X(IR,IT)=D1(IR,IT)
22     CONTINUE
21     CONTINUE
74     IF(N2.EQ.0) GO TO 75
       M1=N1+1
       DO 25 IA=M1,N2
       DO 26 JA=1,N
       X(IA,JA)=D2((IA-N1),JA)
26     CONTINUE
25     CONTINUE
75     IF(N3.EQ.0) GO TO 76
       M2=N1+N2+1
       DO 27 IAA=M2,N3
       DO 28 JAA=1,N
       X(IAA,JAA)=D3((IAA-N1-N2),JAA)
28     CONTINUE
27     CONTINUE
76     IF(N4.EQ.0) GO TO 77
       M3=N1+N2+N3+1
       DO 29 IBB=M3,N4
       DO 30 JBB=1,N
       X(IBB,JBB)=D4((IBB-N1-N2-N3),JBB)
30     CONTINUE
29     CONTINUE
77     IF(N5.EQ.0) GO TO 120
       M4=N1+N2+N3+N4+1
       DO 113 IJIG=M4,N5
       DO 114 JJIG=1,N
       X(IJIG,JJIG)=D5((IJIG-M4+1),JJIG)
114    CONTINUE
113    CONTINUE
120    CALL WTEST(KK,NN,X,US,DIFF,IPR,AMAT,AINVER,
      +TEMPD,RMAX,PMAT,QBAR,SUMSQ,A,YBAR,SW)
       DO 91 IZIP=1,9
       IF(NP.NE.2) GO TO 90
       IF(SW.GT.AK2(N,IZIP)) GO TO 91
       POW(IZIP)=POW(IZIP)+1.0
       GO TO 91
90     IF(NP.NE.3) GO TO 93
       IF(SW.GT.AK3(N,IZIP)) GO TO 91
       POW(IZIP)=POW(IZIP)+1.0
       GO TO 91
93     IF(NP.NE.4) GO TO 91
       IF(SW.GT.AK4(N,IZIP)) GO TO 91
       POW(IZIP)=POW(IZIP)+1.0
91     CONTINUE
       DO 106 I9=1,10
       DO 107 J9=1,100
       X(I9,J9)=0.0
107    CONTINUE
106    CONTINUE
       SW=0.0
78     CONTINUE
```

```
      DO 98 IRIZ=1,9
      POWER(IRIZ)=POW(IRIZ)/FLOAT(NTOT)
98    CONTINUE
      WRITE(6,94)
94    FORMAT(///,10X,"MONTE CARLO/MULTIVARIATE SIMULATION EXPERIMENT")
      WRITE(6,95)NP
95    FORMAT(//,20X,"NUMBER OF VARIATES=",I2)
      WRITE(6,96)N1,N2,N3,N4,N5
96    FORMAT(//,4X,"UNIFORM=",I2,2X,"BETA=",I2,2X,"EXPONENTIAL=",I2,2X,
     +"BINOMIAL=",I2,2X,"NORMAL=",I2)
      WRITE(6,100)
100   FORMAT(///,9X,"EMPIRICALLY DERIVED POWERS FOR THE W* STATISTIC")
      WRITE(6,105)NTOT
105   FORMAT(/,20X,"NUMBER OF SAMPLES TAKEN=",I3)
      WRITE(6,101)NP,N
101   FORMAT(/,26X,"K=",I2," , N=",I2)
      WRITE(6,102)
102   FORMAT(//,4X,".01",6X,".02",6X,".05",6X,".10",6X,
     +".50",6X,".90",6X,".95",6X,".98",6X,".99")
      WRITE(6,99)(POWER(JWT),JWT=1,9)
99    FORMAT(//,1X,F8.6,1X,F8.6,1X,F8.6,1X,F8.6,1X,F8.6,1X,F8.6,
     +1X,F8.6,1X,F8.6,1X,F8.6)
      WRITE(15,939)N,(POWER(IWT),IWT=1,9)
939   FORMAT(I2,4X,F6.3,2X,F6.3,2X,F6.3,2X,F6.3,2X,F6.3,
     +2X,F6.3,2X,F6.3,2X,F6.3,2X,F6.3)
      READ(5,*)NUT
      IF(NUT.EQ.0) GO TO 30
      STOP
      END
```

```
      SUBROUTINE WTEST(KK,NN,X,US,DIFF,IPR,AMAT,AINVER,
     +TEMPD,RMAX,PMAT,QBAR,SUMSQ,A,YBAR,SW)
      DIMENSION X(10,30),US(30),A(30),YBAR(10),DIFF(10,30),
     +IPR(30),AMAT(10,10),AINVER(10,10),TEMPD(10,30),RMAX(30),
     +PMAT(10,10),QBAR(10),SUMSQ(10,10)
      SW=0.0
      SUM8=0.0
      DO 30 J=1,KK
      DO 31 K=1,NN
      US(K)=0.
      RMAX(K)=0.
      TEMPD(J,K)=0.
31    DIFF(J,K)=0.
30    CONTINUE
      DO 25 J=1,KK
      DO 26 K=1,KK
      AINVER(J,K)=0.
26    SUMSQ(J,K)=0.
25    CONTINUE
      SUM7=0.
      DO 2 J=1,KK
      SUM6=0.
      DO 11 K=1,NN
      SUM6=SUM6+X(J,K)
11    CONTINUE
      YBAR(J)=SUM6/FLOAT(NN)
2     CONTINUE
      DO 33 LL=1,KK
83    QBAR(LL)=YBAR(LL)
      DO 12 K=1,NN
      DO 13 J=1,KK
13    DIFF(J,K)=X(J,K)-YBAR(J)
      DO 14 L=1,KK
      DO 15 LL=1,KK
15    SUMSQ(L,LL)=SUMSQ(L,LL)+(DIFF(L,K)*DIFF(LL,K))
14    CONTINUE
12    CONTINUE
      DO 60 J=1,KK
      DO 70 K=1,KK
      PMAT(J,K)=SUMSQ(J,K)
70    AMAT(J,K)=SUMSQ(J,K)
60    CONTINUE
      CALL INVERS(SUMSQ,10,KK,IPR,AINVER,D1)
      DO 16 K=1,NN
      DO 99 J=1,KK
      DO 38 JJ=1,KK
38    TEMPD(J,K)=TEMPD(J,K)+(DIFF(JJ,K)*AINVER(JJ,J))
99    CONTINUE
16    CONTINUE
19    DO 20 K=1,NN
      DO 21 J=1,KK
21    RMAX(K)=RMAX(K)+(TEMPD(J,K)*DIFF(J,K))
20    CONTINUE
      SMAX=RMAX(1)
      DO 22 K=2,NN
      IF(RMAX(K).GE.SMAX)GO TO 40
      GO TO 22
40    SMAX=RMAX(K)
      MAX=K
22    CONTINUE
      DO 23 J=1,NN
      DO 24 K=1,KK
24    US(J)=US(J)+(TEMPD(K,MAX)*DIFF(K,J))
23    CONTINUE
```

```
      L=NN-1
      DO 50 J=1,L
      IP1=J+1
      DO 69 K=IP1,NN
      IF(US(J).LE.US(K))GO TO 69
      TEMP1=US(J)
      US(J)=US(K)
      US(K)=TEMP1
69    CONTINUE
50    CONTINUE
      DO 68 J=1,NN
      SUM7=SUM7+(A(J)*US(J))
68    CONTINUE
      IF(SUM7.LE.0.00001.AND.RMAX(MAX).LE.0.00001) GO TO 900
      SW=(SUM7**2)/RMAX(MAX)
      GO TO 999
900   SW=1.0
999   RETURN
      END
```

```
      PROGRAM SKEWKUR(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE10,
     +TAPE11,TAPE12)
      DIMENSION R(10),X(10,30),D1(5,30),D2(5,30),D3(5,30),
     +D4(5,30),D5(5,30),XBAR(10),DIF(10,30),
     +IPR(30),S(10,10),SINVER(10,10),TEMP(10),
     +B(10,10),XZM(10),TX(30,10),BSKEW(500),NBR(6),
     +BKUR(500),POW(9),POWER(9),HOLD(10),ALPHA(10),BCRIT(10),
     +BMIN(10),BMAX(10)
80    READ(5,*)NP,N,N1,N2,N3,N4,N5,ISEED,NTOT,P,Q,XM,NB,PB
      PRINT(6,*)NP,N,N1,N2,N3,N4,N5,ISEED,NTOT,P,Q,XM,NB,PB
      ALPHA(1)=.01
      ALPHA(2)=.02
      ALPHA(3)=.05
      ALPHA(4)=.10
      ALPHA(5)=.50
      ALPHA(6)=.90
      ALPHA(7)=.95
      ALPHA(8)=.98
      ALPHA(9)=.99
      NN=N**2
      DO 15 I=1,9
      DO 20 J=1,100
      X(I,J)=0.0
      POW(I)=0.0
20    CONTINUE
15    CONTINUE
      DO 76 IKEEP=1,NTOT
      IF(N1.EQ.0)GO TO 70
      DO 81 IM=1,N1
      CALL GGUB(ISEED,N,R)
      DO 82 JM=1,N
      D1(IM,JM)=R(JM)
82    CONTINUE
81    CONTINUE
70    IF(N2.EQ.0) GO TO 71
      DO 83 ISW=1,N2
      CALL GGBTA(ISEED,P,Q,N,R)
      DO 84 JSW=1,N
      D2(ISW,JSW)=R(ISW)
84    CONTINUE
83    CONTINUE
71    IF(N3.EQ.0) GO TO 72
      DO 85 IYT=1,N3
      CALL GGEXP(ISEED,XM,N,R)
      DO 86 JYT=1,N
      D3(IYT,JYT)=R(IYT)
86    CONTINUE
85    CONTINUE
72    IF(N4.EQ.0) GO TO 73
      DO 87 IOP=1,N4
      DO 88 JOP=1,N
      D4(IOP,JOP)=GGBIN(ISEED,NB,PB)
88    CONTINUE
87    CONTINUE
73    IF(N5.EQ.0) GO TO 110
      DO 111 IMAR=1,N5
      CALL GGNOR(ISEED,N,R)
      DO 112 JMAR=1,N
      D5(IMAR,JMAR)=R(JMAR)
112   CONTINUE
111   CONTINUE
110   IF(N1.EQ.0) GO TO 74
      DO 21 IR=1,N1
      DO 22 IT=1,N
```

```
      X(IR,IT)=D1(IR,IT)
22    CONTINUE
21    CONTINUE
74    IF(N2.EQ.0) GO TO 75
      M1=N1+1
      DO 25 IA=M1,N2
      DO 26 JA=1,N
      X(IA,JA)=D2((IA-N1),JA)
26    CONTINUE
25    CONTINUE
75    IF(N3.EQ.0) GO TO 76
      M2=N1+N2+1
      DO 27 IAA=M2,N3
      DO 28 JAA=1,N
      X(IAA,JAA)=D3((IAA-N1-N2),JAA)
28    CONTINUE
27    CONTINUE
76    IF(N4.EQ.0) GO TO 77
      M3=N1+N2+N3+1
      DO 29 IBB=M3,N4
      DO 30 JBB=1,N
      X(IBB,JBB)=D4((IBB-N1-N2-N3),JBB)
30    CONTINUE
29    CONTINUE
77    IF(N5.EQ.0) GO TO 120
      M4=N1+N2+N3+N4+1
      DO 113 IJIG=M4,N5
      DO 114 JJIG=1,N
      X(IJIG,JJIG)=D5((IJIG-M4+1),JJIG)
114   CONTINUE
113   CONTINUE
120   DO 300 I=1,NP
      SUMX=0.0
      DO 301 J=1,N
      SUMX=SUMX+X(I,J)
301   CONTINUE
      XBAR(I)=SUMX/FLOAT(N)
300   CONTINUE
      DO 302 M=1,N
      DO 303 L=1,NP
      DIF(L,M)=X(L,M)-XBAR(L)
303   CONTINUE
302   CONTINUE
      DO 304 I=1,NP
      DO 305 J=1,N
      TX(J,I)=X(I,J)
305   CONTINUE
304   CONTINUE
      NBR(1)=NP
      NBR(2)=N
      NBR(3)=N
      NBR(4)=1
      NBR(5)=1
      NBR(6)=0
      CALL BECOVM(TX,30,NBR,TEMP,XZM,S,IER)
      CALL VCVTSF(S,NP,B,10)
      NZ=N-1
      CALL INVERS(B,10,NP,IPR,SINVER,DIE)
      SUMKUR=0.0
      SKSUM=0.0
      DO 307 I=1,N
      DO 400 L=1,10
400   HOLD(L)=0.0
      DO 309 M=1,NP
```

```
      DO 310 K=1,NP
      HOLD(M)=HOLD(M)+DIF(K,I)*SINVER(K,M)
310   CONTINUE
309   CONTINUE
      DO 308 J=1,N
      SUY=0.0
      DO 311 II=1,NP
      SUY=SUY+HOLD(II)*DIF(II,J)
311   CONTINUE
      SKSUM=SKSUM+SUY**3.0
308   CONTINUE
      SKUR=0.0
      DO 312 LL=1,NP
      SKUR=SKUR+HOLD(LL)*DIF(LL,I)
312   CONTINUE
      SUMKUR=SUMKUR+SKUR**2.0
307   CONTINUE
      BSKEW(IKEEP)=SKSUM/FLOAT(NN)
      BKUR(IKEEP)=SUMKUR/FLOAT(N)
78    CONTINUE
      DO 700 IQ=1,NTOT
      DO 701 JQ=IQ,NTOT
      IF(BSKEW(IQ).GT.BSKEW(JQ)) GO TO 705
      BT=BSKEW(IQ)
      BSKEW(IQ)=BSKEW(JQ)
      BSKEW(JQ)=BT
705   IF(BKUR(IQ).GT.BKUR(JQ)) GO TO 701
      BZT=BKUR(IQ)
      BKUR(IQ)=BKUR(JQ)
      BKUR(JQ)=BZT
701   CONTINUE
700   CONTINUE
      DO 851 IC=1,9
      ICOM=INT(ALPHA(IC)*FLOAT(NTOT))
      IKUR=INT(.5*(ALPHA(IC)*FLOAT(NTOT)))
      BCRIT(IC)=BSKEW(ICOM)
      BMAX(IC)=BKUR(IKUR)
      BMIN(IC)=BKUR(NTOT+1-IKUR)
851   CONTINUE
      WRITE(6,852)
852   FORMAT(///,18X,"MULTIVARIATE TESTS FOR SKEWNESS AND KURTOSIS")
      WRITE(6,853)
853   FORMAT(//,7X,"EMPIRICALLY DERIVED CRITICAL VALUES "
     +"FOR B1,P AND B2,P STATISTICS")
      WRITE(6,854)NP,N
854   FORMAT(//,35X,"P=",I2," , N=",I2)
      WRITE(6,855)NTOT
855   FORMAT(//,27X,"NUMBER OF SAMPLES TAKEN=",I3)
      WRITE(6,856)
856   FORMAT(//,33X,"B1,P(SKEWNESS)",/,29X,"UPPER "
     +"PERCENTAGE POINTS")
      WRITE(6,857)
857   FORMAT(/,4X,".01",6X,".02",6X,".05",6X,".10",6X,
     +".50",6X,".90",6X,".95",6X,".98",6X,".99")
      WRITE(6,858)(BCRIT(JWT),JWT=1,9)
858   FORMAT(/,1X,F8.5,1X,F8.5,1X,F8.5,1X,F8.5,1X,F8.5,1X,F8.5,
     +1X,F8.5,1X,F8.5,1X,F8.5)
      WRITE(6,859)
859   FORMAT(//,33X,"B2,P(KURTOSIS)",/,25X,"UPPER"
     +" & LOWER PERCENTAGE POINTS")
      WRITE(6,857)
      WRITE(6,860)
860   FORMAT(/,38X,"UPPER")
      WRITE(6,858)(BMAX(ID),ID=1,9)
```

```
      WRITE(6,861)
861   FORMAT(/,38X,"LOWER")
      WRITE(6,858)(BMIN(IE),IE=1,9)
      WRITE(10,862)NP,N
862   FORMAT(2I2)
      WRITE(10,863)(BCRIT(IS),IS=1,9)
863   FORMAT(9F8.5)
      WRITE(11,862)NP,N
      WRITE(11,863)(BMAX(MS),MS=1,9)
      WRITE(12,862)NP,N
      WRITE(12,863)(BMIN(NS),NS=1,9)
      READ(5,*)NICE
      IF(NICE.EQ.0) GO TO 80
      STOP
      END
```

```
      PROGRAM SKEWKUR(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE10,
     +TAPE11,TAPE12,TAPE13,TAPE14)
      DIMENSION R(10),X(10,30),D1(5,30),D2(5,30),D3(5,30),
     +D4(5,30),D5(5,30),XBAR(10),DIF(10,30),
     +IPR(30),S(10,10),SINVER(10,10),TEMP(10),
     +B(10,10),XZM(10),TX(30,10),BSKEW(500),NBR(6),
     +BKUR(500),POW1(9),POWER1(9),HOLD(10),ALPHA(10),POW2(10),
     +POWER2(9),Z2(20,10),Z3(20,10),Z4(20,10),Z8(20,10),BZ2(20,10),
     +BZ3(20,10),BZ4(20,10),BZ8(20,10),CZ2(20,10),CZ3(20,10),
     +CZ4(20,10),CZ8(20,10)
933   FORMAT(9F8.5)
932   FORMAT(2I2)
      DO 931 I5=1,10
      READ(10,932)ICRY,IDIE
      READ(10,933)(Z2(IDIE,JEL),JEL=1,9)
      READ(11,932)ICRY,IDIE
      READ(11,933)(BZ2(IDIE,MEL),MEL=1,9)
      READ(12,932)ICRY,IDIE
      READ(12,933)(CZ2(IDIE,NEL),NEL=1,9)
931   CONTINUE
      DO 934 I6=1,9
      READ(10,932)ICRY,IDIE
      READ(10,933)(Z3(IDIE,JEL),JEL=1,9)
      READ(11,932)ICRY,IDIE
      READ(11,933)(BZ3(IDIE,MEL),MEL=1,9)
      READ(12,932)ICRY,IDIE
      READ(12,933)(CZ3(IDIE,NEL),NEL=1,9)
934   CONTINUE
      DO 935 I7=1,9
      READ(10,932)ICRY,IDIE
      READ(10,933)(Z4(IDIE,JEL),JEL=1,9)
      READ(11,932)ICRY,IDIE
      READ(11,933)(BZ4(IDIE,MEL),MEL=1,9)
      READ(12,932)ICRY,IDIE
      READ(12,933)(CZ4(IDIE,NEL),NEL=1,9)
935   CONTINUE
      DO 936 I8=1,6
      READ(10,932)ICRY,IDIE
      READ(10,933)(Z8(IDIE,JEL),JEL=1,9)
      READ(11,932)ICRY,IDIE
      READ(11,933)(BZ8(IDIE,MEL),MEL=1,9)
      READ(12,932)ICRY,IDIE
      READ(12,933)(CZ8(IDIE,NEL),NEL=1,9)
936   CONTINUE
      WRITE(13,521)
521   FORMAT(23X,"EMPIRICALLY DERIVED POWERS FOR B1,P",
     +//,34X,"ALPHA LEVELS")
      WRITE(13,522)
522   FORMAT(8X,".01",5X,".02",5X,".05",5X,".10",5X,
     +".50",5X,".90",5X,".95",5X,".98",5X,".99")
      WRITE(13,523)
523   FORMAT(//,1X,"N",37X,"P=3")
      WRITE(14,524)
524   FORMAT(23X,"EMPIRICALLY DERIVED POWERS FOR B2,P",
     +//,34X,"ALPHA LEVELS")
      WRITE(14,522)
      WRITE(14,523)
80    READ(5,*)NP,N,N1,N2,N3,N4,N5,ISEED,NTOT,P,Q,XM,NB,PB
      PRINT(6,*)NP,N,N1,N2,N3,N4,N5,ISEED,NTOT,P,Q,XM,NB,PB
      ALPHA(1)=.01
      ALPHA(2)=.02
      ALPHA(3)=.05
      ALPHA(4)=.10
      ALPHA(5)=.50
```

```
      ALPHA(6)=.90
      ALPHA(7)=.95
      ALPHA(8)=.98
      ALPHA(9)=.99
      NN=N**2
      DO 15 I=1,9
      DO 20 J=1,100
      X(I,J)=0.0
      POW1(I)=0.0
      POW2(I)=0.0
      POWER1(I)=0.0
      POWER2(I)=0.0
20    CONTINUE
15    CONTINUE
      DO 78 IKEEP=1,NTOT
      IF(N1.EQ.0)GO TO 70
      DO 81 IM=1,N1
      CALL GGUB(ISEED,N,R)
      DO 82 JM=1,N
      D1(IM,JM)=R(JM)
82    CONTINUE
81    CONTINUE
70    IF(N2.EQ.0) GO TO 71
      DO 83 ISH=1,N2
      CALL GGBTA(ISEED,P,Q,N,R)
      DO 84 JSH=1,N
      D2(ISH,JSH)=R(JSH)
84    CONTINUE
83    CONTINUE
71    IF(N3.EQ.0) GO TO 72
      DO 85 IYT=1,N3
      CALL GGEXP(ISEED,XM,N,R)
      DO 86 JYT=1,N
      D3(IYT,JYT)=R(JYT)
86    CONTINUE
85    CONTINUE
72    IF(N4.EQ.0) GO TO 73
      DO 87 IOP=1,N4
      DO 88 JOP=1,N
      D4(IOP,JOP)=GGBIN(ISEED,NB,PB)
88    CONTINUE
87    CONTINUE
73    IF(N5.EQ.0) GO TO 110
      DO 111 IMAR=1,N5
      CALL GGNOR(ISEED,N,R)
      DO 112 JMAR=1,N
      D5(IMAR,JMAR)=R(JMAR)
112   CONTINUE
111   CONTINUE
110    IF(N1.EQ.0) GO TO 74
      DO 21 IR=1,N1
      DO 22 IT=1,N
      X(IR,IT)=D1(IR,IT)
22    CONTINUE
21    CONTINUE
74    IF(N2.EQ.0) GO TO 75
      M1=N1+1
      DO 25 IA=M1,N2
      DO 26 JA=1,N
      X(IA,JA)=D2((IA-N1),JA)
26    CONTINUE
25    CONTINUE
75    IF(N3.EQ.0) GO TO 76
      M2=N1+N2+1
```

```
      DO 27 IAA=M2,N3
      DO 28 JAA=1,N
      X(IAA,JAA)=D3((IAA-N1-N2),JAA)
28    CONTINUE
27    CONTINUE
76    IF(N4.EQ.0) GO TO 77
      M3=N1+N2+N3+1
      DO 29 IBB=M3,N4
      DO 30 JBB=1,N
      X(IBB,JBB)=D4((IBB-N1-N2-N3),JBB)
30    CONTINUE
29    CONTINUE
77    IF(N5.EQ.0) GO TO 120
      M4=N1+N2+N3+N4+1
      DO 113 IJIG=M4,N5
      DO 114 JJIG=1,N
      X(IJIG,JJIG)=D5((IJIG-M4+1),JJIG)
114   CONTINUE
113   CONTINUE
120   DO 300 I=1,NP
      SUMX=0.0
      DO 301 J=1,N
      SUMX=SUMX+X(I,J)
301   CONTINUE
      XBAR(I)=SUMX/FLOAT(N)
300   CONTINUE
      DO 302 M=1,N
      DO 303 L=1,NP
      DIF(L,M)=X(L,M)-XBAR(L)
303   CONTINUE
302   CONTINUE
      DO 304 I=1,NP
      DO 305 J=1,N
      TX(J,I)=X(I,J)
305   CONTINUE
304   CONTINUE
      NBR(1)=NP
      NBR(2)=N
      NBR(3)=N
      NBR(4)=1
      NBR(5)=1
      NBR(6)=0
      CALL BECOVM(TX,30,NBR,TEMP,XZM,S,IER)
      CALL VCVTSF(S,NP,B,10)
      NZ=N-1
      CALL INVERS(B,10,NP,IPR,SINVER,DIE)
      SUMKUR=0.0
      SKSUM=0.0
      DO 307 I=1,N
      DO 400 L=1,10
400   HOLD(L)=0.0
      DO 309 M=1,NP
      DO 310 K=1,NP
      HOLD(M)=HOLD(M)+DIF(K,I)*SINVER(K,M)
310   CONTINUE
309   CONTINUE
      DO 308 J=1,N
      SUY=0.0
      DO 311 II=1,NP
      SUY=SUY+HOLD(II)*DIF(II,J)
311   CONTINUE
      SKSUM=SKSUM+SUY**3.0
308   CONTINUE
      SKUR=0.0
```

```
      DO 312 LL=1,NP
      SKUR=SKUR+HOLD(LL)*DIF(LL,I)
312   CONTINUE
      SUMKUR=SUMKUR+SKUR**2.0
307   CONTINUE
      BSKEW(IKEEP)=SKSUM/FLOAT(NN)
      BKUR(IKEEP)=SUMKUR/FLOAT(N)
78    CONTINUE
      DO 801 IN=1,NTOT
      DO 802 JN=1,9
      IF(NP.NE.2) GO TO 803
      IF(BSKEW(IN).LT.Z2(N,JN)) GO TO 804
      POW1(JN)=POW1(JN)+1.0
804   IF(BKUR(IN).LT.BZ2(N,JN).AND.BKUR(IN).GT.CZ2(N,JN)) GO TO 803
      POW2(JN)=POW2(JN)+1.0
803   IF(NP.NE.3) GO TO 805
      IF(BSKEW(IN).LT.Z3(N,JN)) GO TO 806
      POW1(JN)=POW1(JN)+1.0
806   IF(BKUR(IN).LT.BZ3(N,JN).AND.BKUR(IN).GT.CZ3(N,JN)) GO TO 805
      POW2(JN)=POW2(JN)+1.0
805   IF(NP.NE.4) GO TO 807
      IF(BSKEW(IN).LT.Z4(N,JN)) GO TO 808
      POW1(JN)=POW1(JN)+1.0
808   IF(BKUR(IN).LT.BZ4(N,JN).AND.BKUR(IN).GT.CZ4(N,JN)) GO TO 807
      POW2(JN)=POW2(JN)+1.0
807   IF(NP.NE.8) GO TO 802
      IF(BSKEW(IN).LT.Z8(N,JN)) GO TO 809
      POW1(JN)=POW1(JN)+1.0
809   IF(BKUR(IN).LT.BZ8(N,JN).AND.BKUR(IN).GT.CZ8(N,JN)) GO TO 802
      POW2(JN)=POW2(JN)+1.0
802   CONTINUE
801   CONTINUE
      DO 810 LI=1,9
      POWER1(LI)=POW1(LI)/FLOAT(NTOT)
      POWER2(LI)=POW2(LI)/FLOAT(NTOT)
810   CONTINUE
      WRITE(6,852)
852   FORMAT(///,18X,"MULTIVARIATE TESTS FOR SKEWNESS AND KURTOSIS")
      WRITE(6,853)
853   FORMAT(//,11X,"EMPIRICALLY DERIVED POWERS "
     +"FOR B1,P AND B2,P STATISTICS")
      WRITE(6,854)NP,N
854   FORMAT(//,35X,"P=",I2," , N=",I2)
      WRITE(6,811)N1,N2,N3,N4,N5
811   FORMAT(//,11X,"UNIFORM=",I2,2X,"BETA=",I2,2X,"EXPONENTIAL=",I2,
     +2X,"BINOMIAL=",I2,2X,"NORMAL=",I2)
      WRITE(6,855)NTOT
855   FORMAT(//,27X,"NUMBER OF SAMPLES TAKEN=",I3)
      WRITE(6,856)
856   FORMAT(//,33X,"B1,P(SKEWNESS)")
      WRITE(6,857)
857   FORMAT(/,4X,".01",6X,".02",6X,".05",6X,".10",6X,
     +".50",6X,".90",6X,".95",6X,".98",6X,".99")
      WRITE(6,858)(POWER1(JWT),JWT=1,9)
858   FORMAT(/,1X,F8.5,1X,F8.5,1X,F8.5,1X,F8.5,1X,F8.5,1X,F8.5,
     +1X,F8.5,1X,F8.5,1X,F8.5)
      WRITE(6,859)
859   FORMAT(//,33X,"B2,P(KURTOSIS)")
      WRITE(6,857)
      WRITE(6,858)(POWER2(ID),ID=1,9)
      WRITE(13,863)N,(POWER1(IS),IS=1,9)
863   FORMAT(I2,4X,F6.3,2X,F6.3,2X,F6.3,2X,F6.3,2X,F6.3,
     +2X,F6.3,2X,F6.3,2X,F6.3,2X,F6.3)
      WRITE(14,863)N,(POWER2(MS),MS=1,9)
```

```
READ(5,*)NICE
IF(NICE.EQ.0) GO TO 80
STOP
END
```

APPENDIX B

This appendix contains tables of percentage points for B1,P and B2,P, Table B.1, Table B.2 and Table B.3. Table B.4, reproduced in part from reference 52, contains percentage points for W* derived by Young.

Table B.1

EMPIRICALLY DERIVED CRITICAL VALUES FOR B1,P

ALPHA LEVELS

N	.01	.02	.05	.10	.50	.90	.95	.98	.99
					P=2				
4	1.254	1.242	1.226	1.196	.994	.246	.146	.061	.038
5	2.185	2.332	1.875	1.716	1.634	.326	.241	.154	.138
6	2.546	2.453	2.164	2.017	1.662	.327	.211	.135	.107
7	3.112	2.911	2.600	2.334	1.030	.342	.229	.160	.141
10	3.564	3.127	2.550	2.236	.954	.374	.210	.111	.093
12	4.145	3.632	2.615	2.127	.953	.295	.196	.126	.099
14	4.329	3.278	2.602	2.169	.787	.273	.188	.132	.110
16	3.655	2.965	2.246	1.800	.739	.198	.128	.069	.063
18	3.000	2.520	2.071	1.696	.689	.203	.140	.070	.057
20	2.914	2.556	2.148	1.646	.607	.200	.157	.085	.063
					P=3				
5	3.811	3.800	3.765	3.642	2.931	2.108	1.957	1.851	1.817
6	4.675	4.711	4.569	4.330	3.026	1.940	1.636	1.315	1.182
7	5.744	5.611	5.093	4.637	3.120	1.865	1.511	1.152	1.027
10	5.532	5.565	5.177	4.613	2.819	1.470	1.229	1.007	.796
12	6.091	5.569	4.666	4.199	2.577	1.393	1.124	.921	.867
14	6.544	5.553	5.133	4.454	2.414	1.259	1.074	.754	.581
16	5.737	5.550	4.705	4.021	2.142	1.156	.938	.733	.675
18	5.448	4.957	4.771	3.642	1.982	1.004	.357	.543	.492
20	5.723	5.229	4.051	3.650	2.124	.954	.762	.619	.525
					P=4				
6	8.036	7.471	7.765	7.495	6.251	5.377	5.259	5.108	5.000
7	9.383	9.351	8.348	7.557	5.424	4.970	4.554	4.266	4.194
8	9.950	9.657	8.973	8.375	6.256	4.750	4.407	3.945	3.834
10	10.407	9.807	8.867	8.329	5.929	4.217	3.589	3.139	3.051
12	10.236	9.497	8.697	7.675	5.433	3.691	3.260	2.690	2.464
14	9.533	9.155	8.499	7.699	5.056	3.362	2.994	2.522	2.399
16	8.959	8.302	7.769	6.696	4.653	3.025	2.771	2.404	2.232
18	9.624	8.357	7.317	6.558	4.239	2.700	2.275	1.906	1.758
20	8.302	7.887	6.896	5.062	3.867	2.582	2.187	1.807	1.479
					P=8				
10	41.307	41.353	40.485	39.566	37.661	36.419	36.184	35.929	35.781
12	40.802	39.954	39.077	36.315	34.520	32.056	31.396	30.979	30.505
14	39.741	38.696	37.356	36.264	31.813	29.765	27.896	26.590	26.131
16	37.795	36.971	35.564	33.914	29.539	26.261	25.260	24.527	23.945
18	37.274	35.035	33.581	32.045	27.613	23.642	23.201	22.424	21.705
20	33.790	33.349	32.154	30.514	25.744	21.613	21.010	19.746	19.272

Table B.2

EMPIRICALLY DERIVED CRITICAL VALUES FOR B2,P
(LOWER PERCENTAGE POINTS)

N	.01	.02	.05	.10	.50	.90	.95	.98	.99
					P=2				
4	2.252	2.257	2.280	2.312	2.565	2.716	2.734	2.744	2.750
5	2.665	2.732	2.754	2.816	3.106	3.310	3.335	3.359	3.364
6	2.922	2.975	3.073	3.175	3.549	3.629	3.660	3.902	3.904
7	3.198	3.312	3.412	3.555	3.991	4.361	4.419	4.463	4.468
10	3.590	3.661	3.824	4.069	4.748	5.126	5.198	5.221	5.231
12	4.044	4.215	4.236	4.392	5.015	5.429	5.497	5.567	5.592
14	4.339	4.369	4.575	4.636	5.231	5.731	5.770	5.822	5.825
16	4.435	4.486	4.627	4.860	5.525	5.902	5.983	6.014	6.020
18	4.556	4.675	4.804	5.003	5.682	6.129	6.161	6.211	6.215
20	4.705	4.754	4.899	5.154	5.841	6.284	6.334	6.365	6.369
					P=3				
5	5.873	5.895	5.914	5.949	6.165	6.346	6.359	6.371	6.376
6	6.345	6.462	6.531	6.641	7.001	7.305	7.344	7.360	7.366
7	6.837	6.919	7.004	7.167	7.773	8.130	8.226	8.242	8.246
10	7.831	7.937	8.099	8.332	9.140	9.605	9.716	9.776	9.787
12	8.535	8.723	8.823	9.032	9.792	10.376	10.439	10.492	10.506
14	8.817	8.975	9.218	9.462	10.409	11.005	11.070	11.105	11.147
16	9.201	9.306	9.623	9.867	10.689	11.276	11.373	11.412	11.414
18	9.244	9.427	9.562	9.924	11.013	11.621	11.713	11.746	11.751
20	9.551	9.916	10.079	10.367	11.335	11.948	12.068	12.110	12.128
					P=4				
6	11.205	11.247	11.315	11.501	11.555	11.747	11.775	11.795	11.799
7	11.596	12.022	12.152	12.179	12.772	13.125	13.157	13.170	13.185
8	12.649	12.797	12.944	13.176	13.700	14.103	14.166	14.205	14.211
10	13.722	13.916	13.993	14.264	15.110	15.723	15.790	15.828	15.830
12	14.492	14.743	14.861	15.153	16.139	16.736	16.812	16.856	16.866
14	14.959	14.992	15.448	15.776	16.927	17.706	17.761	17.803	17.814
16	15.441	15.716	16.094	16.421	17.421	18.214	18.308	18.326	18.337
18	16.101	16.426	16.783	17.076	18.013	18.729	18.826	18.885	18.895
20	16.969	16.616	17.069	17.384	18.396	19.144	19.270	19.343	19.349
					P=8				
10	52.065	52.132	52.216	52.290	52.599	52.817	52.852	52.874	52.881
12	54.441	54.664	54.813	55.053	55.915	56.405	56.506	56.541	56.559
14	56.155	56.466	56.844	57.215	58.574	59.192	59.278	59.327	59.334
16	58.155	58.290	58.915	59.189	60.441	61.347	61.466	61.523	61.529
18	59.025	59.357	59.943	60.359	62.301	63.339	63.426	63.501	63.511
20	60.604	60.799	61.372	61.632	63.747	64.731	64.883	64.976	64.989

Table B.3

EMPIRICALLY DERIVED CRITICAL VALUES FOR B2,P
(UPPER PERCENTAGE POINTS)

	ALPHA LEVELS								
	.01	.02	.05	.10	.50	.90	.95	.98	.99
N					P=2				
4	2.999	2.997	2.979	2.950	2.843	2.775	2.761	2.755	2.755
5	4.244	4.190	4.152	4.104	3.740	3.464	3.404	3.383	3.377
6	5.429	5.311	5.109	4.936	4.344	4.010	3.964	3.936	3.924
7	6.501	6.374	5.960	5.678	4.929	4.573	4.516	4.497	4.491
10	8.256	7.933	7.546	7.059	5.916	5.391	5.318	5.255	5.249
12	9.593	8.939	7.982	7.626	6.340	5.763	5.709	5.674	5.674
14	10.224	9.726	9.557	8.319	6.647	5.955	5.887	5.862	5.859
16	10.505	9.923	9.130	8.269	6.594	6.138	6.104	6.054	6.050
18	10.239	9.472	8.931	8.243	6.555	6.383	6.304	6.255	6.243
20	11.425	10.573	9.678	8.755	7.052	6.500	6.460	6.414	6.401
					P=3				
5	7.183	7.149	7.093	6.961	6.618	6.464	6.436	6.430	6.392
6	8.968	8.735	9.603	8.432	7.609	7.456	7.403	7.387	7.376
7	11.034	10.416	10.075	9.658	8.800	8.553	8.297	8.268	8.262
10	13.374	13.091	12.450	12.044	10.657	9.956	9.887	9.824	9.822
12	14.215	13.694	13.265	12.525	11.418	10.698	10.610	10.544	10.533
14	15.743	15.265	14.601	14.043	12.210	11.432	11.368	11.233	11.223
16	15.802	15.553	14.654	14.186	12.380	11.584	11.509	11.450	11.441
18	17.369	16.191	15.213	14.293	12.762	11.923	11.849	11.799	11.789
20	18.064	17.174	15.724	14.968	13.218	12.408	12.253	12.206	12.193
					P=4				
6	13.003	12.927	12.820	12.625	12.161	11.856	11.828	11.819	11.813
7	15.295	15.067	14.740	14.448	13.673	13.275	13.227	13.205	13.201
8	17.098	16.767	16.416	16.052	14.961	14.359	14.303	14.277	14.264
10	20.213	19.377	18.569	18.000	16.700	16.021	15.935	15.898	15.881
12	21.819	20.588	20.133	19.316	17.926	17.032	16.955	16.893	16.892
14	22.926	22.376	21.531	20.956	18.945	18.042	17.960	17.885	17.856
16	24.500	23.383	22.255	21.580	19.470	18.630	18.478	18.43[illegible]	18.393
18	25.376	23.775	23.107	22.349	20.113	19.036	19.013	18.953	18.933
20	24.669	24.216	23.846	22.977	20.573	19.576	19.437	19.399	19.396
					P=8				
10	55.111	54.822	54.498	54.239	53.320	52.950	52.926	52.906	52.899
12	60.607	60.033	59.411	59.032	57.374	56.712	56.649	56.614	56.602
14	66.175	64.311	63.613	62.709	60.739	59.631	59.484	59.395	59.387
16	67.498	66.614	66.148	65.322	63.246	61.927	61.758	61.614	61.599
18	72.226	70.712	69.455	67.511	65.234	63.802	63.649	63.551	63.540
20	73.844	71.465	70.886	69.980	66.732	65.357	65.167	65.114	65.261

Table B.4
EMPIRICALLY DERIVED CRITICAL VALUES FOR W*

	ALPHA LEVELS								
	.01	.02	.05	.10	.50	.90	.95	.98	.99
N					P=2				
4	.6315	.6341	.6469	.6615	.7955	.9495	.9702	.9919	.9952
5	.6058	.6283	.6539	.6802	.8507	.9502	.9643	.9735	.9806
6	.5938	.6310	.6829	.7222	.8770	.9549	.9676	.9797	.9876
7	.6298	.6637	.7071	.7575	.8878	.9564	.9645	.9790	.9814
10	.6696	.6939	.7717	.8062	.9052	.9635	.9698	.9789	.9821
12	.7422	.7765	.8110	.8448	.9233	.9664	.9726	.9821	.9848
14	.7692	.7811	.8259	.8564	.9323	.9685	.9740	.9780	.9813
16	.7924	.8125	.8511	.8770	.9392	.9715	.9767	.9824	.9835
18	.8086	.8292	.8610	.8889	.9484	.9734	.9785	.9833	.9850
20	.8410	.8535	.8755	.8951	.9505	.9750	.9797	.9834	.9866
					P=3				
5	.5543	.5557	.5631	.5737	.6668	.8854	.9225	.9524	.9751
6	.5397	.5577	.5779	.6086	.7668	.9256	.9470	.9718	.9818
7	.5370	.5607	.6148	.6516	.8081	.9255	.9417	.9586	.9695
10	.6349	.6678	.7194	.7598	.8723	.9408	.9553	.9644	.9767
12	.6618	.7207	.7592	.7837	.8871	.9447	.9575	.9689	.9796
14	.7061	.7300	.7751	.8101	.8959	.9519	.9605	.9684	.9731
16	.7467	.7706	.8001	.8342	.9099	.9547	.9629	.9716	.9769
18	.7306	.7731	.8237	.8539	.9210	.9615	.9678	.9735	.9762
20	.7720	.7954	.8299	.8593	.9289	.9650	.9695	.9752	.9793
					P=4				
6	.4975	.4984	.5013	.5088	.5776	.7932	.8859	.9297	.9511
7	.4804	.4878	.5095	.5417	.6610	.8422	.9176	.9563	.9723
8	.4875	.5206	.5574	.5866	.7301	.8693	.9200	.9504	.9669
10	.5317	.5887	.6544	.6881	.6008	.9009	.9346	.9608	.9698
12	.5877	.6363	.6776	.7285	.8320	.9091	.9282	.9567	.9626
14	.6693	.6919	.7414	.7712	.8589	.9211	.9335	.9623	.9756
16	.6812	.7262	.7659	.7947	.8786	.9412	.9533	.9643	.9572
18	.7335	.7507	.7925	.8127	.8851	.9411	.9515	.9649	.9706
20	.7667	.7776	.8012	.8281	.9012	.9476	.9569	.9645	.9725

APPENDIX C

This appendix contains tables of power estimates for B1,P, B2,P and W* against pure variates (uniform, beta, exponential, binomial) for P=2,...,8, N=4,...,20 and α=.01,...,.99.

Table C.1
EMPIRICALLY DERIVED POWERS FOR B1,P
(AGAINST UNIFORM VARIATES)

	ALPHA LEVELS								
N	.01	.02	.05	.10	.50	.90	.95	.98	.99
					P=2				
4	.014	.040	.063	.111	.469	.889	.943	.977	.986
5	.011	.011	.023	.057	.391	.851	.914	.966	.971
6	.017	.023	.043	.071	.377	.866	.920	.966	.974
7	.014	.020	.037	.043	.334	.840	.911	.951	.963
10	.003	.006	.017	.023	.311	.834	.906	.960	.974
12	0.000	0.000	.003	.026	.291	.783	.869	.940	.960
14	0.000	.003	.003	.014	.257	.740	.851	.920	.943
16	0.000	0.000	.006	.009	.246	.806	.889	.960	.963
18	0.000	0.000	.003	.009	.234	.771	.894	.969	.980
20	0.000	0.000	0.000	.006	.271	.754	.823	.923	.957
					P=3				
5	0.000	.003	.026	.100	.437	.854	.917	.963	.977
6	.003	.006	.020	.060	.414	.866	.920	.960	.977
7	.006	.006	.020	.054	.360	.829	.926	.969	.977
10	0.000	.003	.009	.031	.317	.826	.880	.934	.966
12	0.000	.003	.009	.020	.300	.797	.894	.943	.949
14	.003	.006	.009	.017	.246	.751	.834	.937	.957
16	0.000	0.000	0.000	.017	.223	.714	.809	.923	.949
18	0.000	0.000	.003	.023	.251	.734	.820	.960	.986
20	0.000	0.000	0.000	.006	.174	.729	.840	.911	.934
					P=4				
6	0.000	.009	.034	.083	.426	.880	.949	.974	.989
7	0.000	.014	.031	.051	.363	.869	.929	.960	.966
8	.006	.006	.023	.049	.371	.849	.914	.971	.977
10	.006	.009	.023	.029	.294	.809	.920	.966	.969
12	0.000	.006	.009	.037	.303	.780	.900	.969	.980
14	.003	.003	.006	.014	.263	.734	.826	.931	.954
16	.006	.006	.006	.020	.206	.731	.820	.914	.943
18	0.000	.003	.003	.011	.214	.714	.854	.929	.954
20	.003	.003	.009	.017	.166	.646	.777	.903	.957
					P=8				
10	.003	.011	.031	.060	.420	.854	.931	.977	.994
12	.003	.003	.009	.034	.331	.814	.917	.940	.983
14	.003	.006	.017	.049	.323	.809	.877	.963	.991
16	0.000	0.000	.003	.020	.251	.703	.811	.931	.963
18	0.000	0.000	.006	.014	.194	.757	.820	.926	.937
20	.003	.003	.003	.011	.200	.729	.849	.937	.957

Table C.2
EMPIRICALLY DERIVED POWERS FOR B2,P
(AGAINST UNIFORM VARIATES)

ALPHA LEVELS

N	.01	.02	.05	.10	.50	.90	.95	.98	.99
					P=2				
4	[illegible]	[illegible]	.040	.100	.483	.889	.963	.989	.994
5	.014	.028	.037	.063	.450	.857	.946	.983	.989
6	.017	.023	.060	.126	.491	.897	.951	.991	.994
7	.020	.050	.094	.165	.560	.871	.943	.980	.983
10	.014	.020	.049	.169	.671	.931	.971	.994	.994
12	.034	.134	.157	.206	.646	.906	.926	.957	.953
14	.091	.123	.231	.271	.726	.943	.977	.994	.997
16	.103	.126	.217	.346	.771	.966	.997	1.000	1.000
18	.074	.140	.231	.391	.797	.951	.971	.989	.991
20	.140	.171	.257	.480	.643	.974	.991	.997	.997
					P=3				
5	0.000	.014	.040	.117	.551	.900	.931	.986	.994
6	0.000	.031	.060	.111	.471	.880	.960	.983	.997
7	.009	.020	.037	.086	.537	.926	.971	.991	.994
10	.023	.029	.071	.137	.549	.874	.940	.980	.980
12	.063	.106	.137	.229	.623	.923	.969	.989	.997
14	.037	.080	.157	.257	.677	.929	.960	.974	.995
16	.071	.111	.240	.363	.783	.966	.986	.994	.994
18	.057	.114	.157	.274	.766	.963	.974	.991	.991
20	.065	.163	.306	.451	.814	.949	.950	.991	.997
					P=4				
6	.003	.014	.040	.080	.454	.911	.957	.989	.991
7	0.000	.020	.054	.103	.497	.909	.974	.994	1.000
8	.011	.029	.063	.126	.514	.900	.946	.971	.980
10	.020	.051	.074	.149	.600	.900	.946	.977	.986
12	.034	.065	.103	.183	.611	.934	.954	.991	.991
14	.023	.026	.117	.191	.663	.937	.954	.991	.997
16	.037	.077	.174	.247	.677	.923	.969	.980	.989
18	.091	.174	.300	.403	.774	.974	.983	.997	1.000
20	.046	.151	.323	.449	.734	.963	.980	.994	.997
					P=8				
10	.003	.003	.031	.066	.540	.923	.960	.991	.991
12	.005	.023	.037	.063	.497	.869	.937	.966	.986
14	.003	.011	.034	.126	.560	.874	.949	.969	.980
16	.020	.034	.103	.174	.566	.909	.960	.980	.989
18	.009	.023	.063	.129	.614	.931	.963	.994	.994
20	.034	.051	.131	.174	.683	.954	.977	.989	.991

Table C.3
EMPIRICALLY DERIVED POWERS FOR THE W* STATISTIC
(AGAINST UNIFORM VARIATES)

N	.01	.02	.05	.10	.50	.90	.95	.98	.99
					ALPHA LEVELS				
					P=2				
4	.009	.026	.063	.097	.506	.931	.960	.994	.994
5	.006	.014	.031	.057	.394	.683	.951	.963	.990
6	.003	.011	.034	.063	.411	.911	.943	.989	.997
7	.011	.017	.029	.060	.414	.914	.946	.986	.989
10	.003	.003	.017	.031	.371	.897	.926	.971	.991
12	0.000	.003	.003	.031	.380	.874	.931	.987	.991
14	.003	.003	.020	.037	.366	.846	.911	.951	.971
16	0.000	.003	.011	.040	.346	.831	.911	.957	.960
18	.006	.009	.011	.049	.437	.663	.923	.960	.960
20	.003	.003	.011	.037	.391	.654	.911	.963	.994
					P=3				
5	.029	.037	.063	.126	.491	.891	.926	.960	.986
6	.011	.034	.060	.094	.517	.920	.963	.983	.994
7	.003	.006	.060	.094	.466	.906	.937	.971	.989
10	0.000	.006	.034	.060	.434	.871	.934	.963	.994
12	0.000	0.000	.014	.031	.383	.869	.931	.980	.997
14	.006	.009	.023	.040	.306	.803	.923	.966	.986
16	0.000	.006	.009	.034	.283	.763	.874	.949	.980
18	0.000	.003	.014	.043	.251	.794	.869	.929	.957
20	0.000	0.000	.006	.017	.254	.797	.886	.951	.980
					P=4				
5	.011	.017	.040	.109	.489	.900	.937	.960	.969
7	.011	.014	.029	.083	.417	.866	.926	.974	.991
8	.009	.029	.057	.094	.434	.849	.917	.963	.977
10	0.000	.020	.049	.083	.411	.854	.926	.980	.983
12	.003	.006	.014	.049	.366	.820	.911	.971	.986
14	.011	.017	.037	.057	.343	.774	.874	.977	.991
16	.003	.011	.014	.026	.300	.843	.937	.971	.980
18	.003	.009	.026	.046	.237	.774	.871	.963	.983
20	.006	.011	.014	.020	.254	.737	.849	.931	.983

Table C.4
EMPIRICALLY DERIVED POWERS FOR B1,P
(AGAINST BETA VARIATES)

	ALPHA LEVELS								
N	.01	.02	.05	.10	.50	.90	.95	.98	.99
					P=2				
4	.012	.029	.071	.111	.523	.943	.986	.997	1.000
5	.026	.031	.060	.109	.546	.903	.934	.974	.977
6	.034	.040	.094	.140	.546	.911	.960	.989	.991
7	.031	.049	.077	.114	.534	.914	.960	.977	.980
10	.033	.126	.200	.260	.677	.960	.983	.989	.989
12	.037	.063	.200	.309	.731	.969	.983	.989	.991
14	.043	.134	.194	.314	.823	.977	.991	.997	.997
16	.077	.154	.311	.454	.869	.986	1.000	1.000	1.000
18	.166	.291	.403	.540	.891	.994	.997	1.000	1.000
20	.211	.283	.400	.580	.934	.997	1.000	1.000	1.000
					P=3				
5	.009	.014	.037	.126	.529	.857	.929	.971	.983
6	.017	.023	.046	.074	.540	.923	.960	.980	.986
7	.026	.026	.086	.137	.520	.920	.957	.991	.994
10	.034	.069	.134	.209	.669	.954	.977	.989	1.000
12	.077	.109	.200	.303	.763	.974	.986	.994	.997
14	.060	.094	.160	.260	.791	.977	.983	.997	.997
16	.140	.180	.254	.374	.843	.991	.994	1.000	1.000
18	.160	.231	.306	.480	.880	.991	.991	1.000	1.000
20	[illegible]	.234	.391	.497	.877	.986	.994	.997	.997
					P=4				
6	.023	.034	.069	.126	.531	.940	.969	.986	.989
7	.014	.023	.074	.137	.531	.920	.963	.989	.991
8	.029	.037	.063	.143	.657	.940	.971	.986	.989
10	.034	.046	.129	.189	.689	.937	.974	.986	.991
12	.031	.077	.154	.274	.720	.959	.997	1.000	1.000
14	.091	.120	.157	.226	.757	.974	.986	1.000	1.000
16	.154	.231	.280	.426	.803	.989	.989	.994	.994
18	.097	.197	.326	.463	.877	.986	.991	.994	.994
20	.171	.231	.351	.500	.909	.991	1.000	1.000	1.000
					P=8				
10	.023	.029	.069	.120	.560	.909	.957	.980	.989
12	.034	.071	.111	.160	.640	.951	.971	.989	.997
14	.051	.071	.129	.200	.711	.951	.974	.997	.997
16	.074	.097	.160	.246	.711	.957	.977	.986	.991
18	.049	.103	.171	.266	.674	.954	.966	.997	1.000
20	.114	.137	.189	.280	.734	.969	.991	1.000	1.000

Table C.5
EMPIFICALLY DERIVED POWERS FOR B2,P
(AGAINST BETA VARIATES)

	ALPHA LEVELS								
	.01	.02	.05	.10	.50	.90	.95	.98	.99
N					P=2				
4	0.000	.006	.026	.069	.460	.663	.931	.969	.983
5	.006	.043	.057	.109	.477	.854	.940	.986	.991
6	.014	.034	.069	.137	.491	.857	.909	.974	.986
7	.020	.051	.091	.203	.551	.877	.946	.974	.986
10	.031	.060	.091	.174	.674	.900	.960	.991	.997
12	.023	.049	.103	.157	.577	.883	.926	.954	.966
14	.031	.057	.114	.177	.620	.934	.963	.986	.989
16	.017	.034	.069	.151	.663	.909	.946	.980	.980
18	.051	.086	.146	.271	.643	.929	.963	.986	.991
20	.037	.057	.129	.243	.729	.960	.989	.994	.994
					P=3				
5	0.000	.011	.040	.111	.560	.846	.917	.974	.986
6	.006	.029	.043	.077	.434	.891	.966	.980	.986
7	.009	.023	.051	.100	.500	.937	.963	.983	.991
10	.006	.014	.074	.169	.497	.869	.929	.977	.980
12	.040	.066	.094	.137	.537	.911	.963	.989	.994
14	.026	.051	.100	.157	.554	.897	.946	.969	.980
16	.066	.071	.146	.214	.603	.917	.966	.994	.994
18	.026	.071	.134	.254	.651	.934	.977	.989	.994
20	.023	.054	.134	.231	.646	.903	.946	.983	.986
					P=4				
6	.017	.029	.049	.089	.451	.906	.949	.977	.986
7	[illegible]	.069	.043	.109	.480	.917	.969	.989	.991
8	.009	.023	.049	.091	.511	.891	.946	.966	.966
10	.017	.040	.069	.143	.540	.914	.957	.980	.991
12	.006	.037	.066	.157	.546	.940	.966	.986	.989
14	.029	.051	.063	.137	.549	.917	.963	.990	.991
16	.026	.077	.151	.217	.606	.934	.980	.991	.997
18	.031	.083	.129	.203	.623	.940	.960	.989	.994
20	.066	.097	.140	.206	.623	.909	.960	.977	.989
					P=8				
10	.017	.029	.054	.089	.511	.909	.943	.971	.983
12	.021	.040	.089	.109	.549	.909	.960	.980	.980
14	.006	.031	.063	.123	.569	.923	.966	.989	.991
16	.029	.077	.111	.160	.554	.909	.957	.977	.980
18	.014	.037	.077	.140	.583	.934	.977	1.000	1.000
20	.029	.080	.109	.154	.651	.911	.951	.974	.989

Table C.6
EMPIRICALLY DERIVED POWERS FOR THE W* STATISTIC
(AGAINST BETA VARIATES)

	ALPHA LEVELS								
	.01	.02	.05	.10	.50	.90	.95	.98	.99
N					P=2				
4	.009	.011	.063	.100	.534	.911	.924	.969	.980
5	.009	.034	.077	.111	.560	.917	.949	.971	.986
6	.017	.034	.094	.154	.577	.923	.951	.980	.994
7	.009	.020	.080	.154	.574	.906	.934	.980	.980
10	.031	.049	.143	.211	.617	.946	.971	.989	1.000
12	.057	.134	.230	.306	.771	.971	.977	.994	.994
14	.060	.069	.197	.329	.783	.954	.980	.994	.994
16	.060	.140	.253	.469	.809	.971	.989	1.000	1.000
18	.129	.163	.334	.463	.871	.989	1.000	1.000	1.000
20	.209	.246	.366	.517	.906	.997	1.000	1.000	1.000
					P=3				
5	.005	.017	.063	.117	.554	.929	.969	.986	.991
6	.011	.037	.063	.123	.574	.934	.954	.980	.994
7	.009	.023	.103	.177	.609	.923	.951	.963	.986
10	.051	.069	.143	.263	.700	.963	.980	.997	1.000
12	.049	.111	.183	.251	.729	.954	.974	.986	.997
14	.063	.100	.180	.297	.731	.977	.986	.994	.997
16	.074	.134	.220	.366	.754	.960	.983	.991	.994
18	.037	.123	.280	.429	.777	.960	.980	.989	.994
20	.109	.183	.274	.426	.829	.971	.983	.994	.997
					P=4				
6	.009	.011	.029	.063	.483	.891	.934	.954	.974
7	.009	.014	.040	.111	.546	.914	.969	.986	.989
8	.009	.054	.097	.151	.596	.889	.949	.974	.994
10	.020	.071	.174	.266	.651	.926	.963	.991	.994
12	.029	.057	.117	.269	.694	.937	.963	.986	.994
14	.071	.097	.200	.297	.740	.943	.963	.989	1.000
16	.063	.123	.209	.320	.760	.960	.960	.963	.971
18	.100	.131	.271	.371	.754	.951	.957	.980	.986
20	.149	.177	.243	.369	.806	.960	.974	.980	.991

Table C.7
EMPIRICALLY DERIVED POWERS FOR B1,P
(AGAINST EXPONENTIAL VARIATES)

	ALPHA LEVELS								
	.01	.02	.05	.10	.50	.90	.95	.98	.99
N					P=2				
4	.043	.074	.109	.151	.577	.949	[illegible]	.997	1.000
5	.017	.020	.054	.131	.586	.911	.951	.959	.991
6	.086	.117	.203	.257	.657	.966	.991	.997	.997
7	.071	.083	.157	.229	.694	.951	.974	[illegible]	.989
10	.197	.311	.434	.506	.806	.983	1.000	1.000	1.000
12	.209	.289	.471	.594	.874	.991	.994	1.000	1.000
14	.257	.430	.480	.666	.969	1.000	1.000	1.000	1.000
16	.371	.503	.671	.769	.963	1.000	1.000	1.000	1.000
18	.583	.700	.794	.860	.969	.997	1.000	1.000	1.000
20	.626	.700	.769	.874	.989	.997	.997	1.000	1.000
					P=3				
5	.009	.023	.046	.126	.554	.917	.977	.991	.997
6	.034	.060	.106	.160	.640	.946	.980	.986	.989
7	.046	.060	.163	.257	.666	.951	.974	.994	.994
10	.149	.269	.380	.494	.846	.994	.997	1.000	1.000
12	.306	.383	.506	.646	.914	.997	1.000	1.000	1.000
14	.331	.431	.540	.669	.951	.986	.994	1.000	1.000
16	.446	.506	.611	.737	.983	1.000	1.000	1.000	1.000
18	.571	.649	.760	.860	.980	1.000	1.000	1.000	1.000
20	.583	.671	.811	.866	.986	1.000	1.000	1.000	1.000
					P=4				
6	.020	.029	.074	.177	.643	.949	.977	.989	.994
7	.014	.037	.137	.223	.631	.957	.983	.989	.989
8	.037	.063	.137	.266	.783	.971	.983	.997	.997
10	.126	.177	.300	.400	.869	.989	1.000	1.000	1.000
12	.234	.326	.446	.574	.920	1.000	1.000	1.000	1.000
14	.411	.463	.557	.657	.957	.997	.997	1.000	1.000
16	.537	.606	.666	.803	.969	1.000	1.000	1.000	1.000
18	.480	.614	.774	.860	.994	1.000	1.000	1.000	1.000
20	.660	.714	.837	.909	.991	1.000	1.000	1.000	1.000
					P=8				
10	.037	.069	.146	.211	.623	.931	.963	.991	.997
12	.103	.149	.231	.309	.811	.986	.989	.997	.997
14	.186	.246	.340	.431	.886	.989	.991	.997	1.000
16	.311	.357	.469	.609	.889	.991	.997	1.000	1.000
18	.320	.486	.594	.731	.954	.991	1.000	1.000	1.000
20	.586	.603	.674	.777	.960	1.000	1.000	1.000	1.000

Table C.8
EMPIRICALLY DERIVED POWERS FOR B2,P
(AGAINST EXPONENTIAL VARIATES)

N	.01	.02	.05	.10	.50	.90	.95	.98	.99
					P=2				
4	0.000	0.000	.014	.063	.429	.880	.940	.966	.991
5	.006	.023	.034	.071	.440	.850	.937	.977	.986
6	.031	.049	.106	.151	.489	.871	.929	.986	.991
7	.026	.046	.100	.186	.571	.886	.943	.977	.986
10	.114	.146	.203	.320	.703	.940	.983	.991	.994
12	.109	.169	.297	.374	.706	.931	.954	.977	.980
14	.137	.174	.263	.343	.743	.940	.974	.989	.989
16	.151	.211	.300	.449	.797	.937	.963	.983	.991
18	.249	.369	.460	.566	.857	.969	.960	.997	1.000
20	.217	.263	.360	.486	.831	.974	.980	.997	.997
					P=3				
5	0.000	0.000	.006	.060	.546	.886	.931	.980	.983
6	.006	.017	.054	.120	.466	.897	.960	.983	.994
7	.006	.034	.080	.134	.560	.914	.954	.991	.991
10	.069	.091	.189	.266	.671	.903	.951	.989	.991
12	.160	.231	.303	.400	.691	.943	.966	1.000	1.000
14	.186	.220	.311	.403	.751	.920	.946	.971	.986
16	.294	.334	.437	.491	.789	.960	.989	1.000	1.000
18	.251	.371	.469	.569	.849	.963	.989	.997	.997
20	.289	.369	.491	.589	.849	.957	.974	.989	.991
					P=4				
6	.017	.023	.051	.089	.526	.903	.963	.989	.994
7	.006	.014	.051	.103	.506	.920	.960	.983	.983
8	.011	.029	.060	.117	.543	.897	.931	.969	.980
10	.023	.054	.191	.263	.657	.943	.971	.983	.986
12	.077	.186	.251	.386	.717	.971	.989	.997	1.000
14	.174	.269	.357	.429	.769	.954	.974	.994	.997
16	.177	.269	.460	.537	.849	.954	.986	.989	.991
18	.251	.397	.480	.583	.823	.977	.989	.994	.994
20	.406	.454	.491	.577	.866	.980	.997	1.000	1.000
					P=8				
10	.014	.034	.080	.134	.554	.909	.943	.969	.980
12	.034	[illegible]	.123	.171	.614	.911	.963	.986	.991
14	.040	.123	.183	.306	.677	.934	.986	.991	.991
16	[illegible]	.274	.334	.417	.743	.943	.971	.991	.991
18	.157	.217	.323	.485	.817	.963	.983	.997	.997
20	.260	.434	.486	.563	.869	.977	.991	.994	.994

Table C.9

EMPIRICALLY DERIVED POWERS FOR THE W* STATISTIC

(AGAINST EXPONENTIAL VARIATES)

	ALPHA LEVELS								
N	.01	.02	.05	.10	.50	.90	.95	.98	.99
					P=2				
4	.009	.020	.083	.134	.586	.940	.966	.997	.997
5	.037	.069	.097	.134	.574	.931	.960	.966	.966
6	.031	.069	.149	.234	.657	.937	.969	.991	.991
7	.066	.100	.159	.251	.703	.966	.980	.991	.994
10	.165	.149	.326	.446	.759	.974	.983	.989	.991
12	.243	.337	.486	.597	.857	.977	.989	1.000	1.000
14	.309	.346	.536	.643	.923	.997	.997	1.000	1.000
16	.331	.426	.614	.749	.946	.991	1.000	1.000	1.000
18	.474	.563	.689	.820	.957	.994	.997	1.000	1.000
20	.580	.623	.746	.837	.971	.994	1.000	1.000	1.000
					P=3				
5	.017	.031	.083	.149	.577	.917	.957	.971	.991
6	.031	.069	.146	.220	.677	.951	.977	.994	.994
7	.054	.083	.194	.277	.686	.940	.969	.989	.994
10	.129	.231	.366	.474	.826	.943	.977	.983	.994
12	.174	.311	.431	.529	.863	.974	.980	.994	1.000
14	.251	.326	.457	.574	.877	.977	.989	.994	.997
16	.337	.414	.483	.620	.914	.991	.997	.997	.997
18	.309	.426	.643	.766	.940	.997	1.000	1.000	1.000
20	.406	.517	.666	.780	.951	.989	.994	1.000	1.000
					P=4				
6	.008	.014	.043	.069	.597	.937	.960	.980	.989
7	.026	.040	.091	.165	.553	.839	.963	.989	.989
8	.040	.080	.174	.263	.691	.897	.951	.977	.991
10	.063	.183	.354	.457	.783	.929	.960	.986	1.000
12	.111	.217	.374	.517	.843	.954	.983	.994	.994
14	.309	.351	.483	.606	.900	.960	.971	.991	1.000
16	.280	.397	.543	.669	.926	.974	.983	.991	.994
18	.437	.474	.657	.746	.920	.971	.974	.986	.989
20	.491	.549	.640	.769	.954	.989	.989	.991	.991

Table C.10
EMPIRICALLY DERIVED POWERS FOR B1,P
(AGAINST BINOMIAL VARIATES)

	ALPHA LEVELS								
	.01	.02	.05	.10	.50	.90	.95	.98	.99
N					P=2				
4	.043	.051	.069	.103	.471	.897	.954	.986	.991
5	.009	.011	.040	.089	.503	.909	.937	.977	.977
6	.020	.026	.069	.111	.500	.877	.957	.977	.989
7	.009	.017	.031	.051	.454	.894	.943	.974	.983
10	.009	.017	.046	.080	.489	.886	.943	.989	.994
12	.011	.011	.074	.106	.463	.874	.937	.977	.980
14	0.000	.009	.014	.051	.511	.874	.931	.963	.974
16	.009	.009	.043	.083	.460	.914	.960	.980	.985
18	.011	.023	.060	.123	.443	.874	.946	.994	.997
20	.014	.023	.037	.097	.511	.880	.914	.977	.989
					P=3				
5	.014	.020	.043	.143	.477	.891	.951	.983	.991
6	.011	.040	.066	.106	.494	.920	.966	.980	.989
7	0.000	0.000	.037	.083	.457	.891	.949	.966	.974
10	.009	.026	.057	.103	.500	.889	.937	.971	.994
12	.014	.026	.051	.131	.506	.894	.960	.983	.989
14	.006	.017	.029	.063	.469	.894	.937	.971	.980
16	.020	.029	.043	.080	.497	.857	.926	.977	.980
18	.014	.020	.043	.083	.520	.889	.940	.991	.994
20	0.000	.006	.023	.043	.369	.889	.954	.986	.989
					P=4				
6	.003	.017	.060	.137	.509	.903	.957	.983	.989
7	.006	.014	.066	.123	.474	.894	.954	.974	.974
8	.011	.020	.037	.066	.500	.871	.929	.974	.977
10	.009	.014	.037	.077	.474	.877	.949	.963	.966
12	0.000	.006	.023	.091	.517	.906	.954	.959	.997
14	.009	.026	.051	.077	.469	.869	.934	.971	.981
16	.020	.037	.054	.120	.463	.883	.926	.971	.977
18	0.000	.020	.037	.083	.503	.926	.969	.993	.993
20	.006	.006	.026	.083	.517	.903	.957	.959	.997
					P=8				
10	.009	.020	.051	.094	.457	.906	.963	.986	.991
12	.006	.014	.031	.066	.549	.900	.974	.983	.989
14	0.000	.006	.057	.086	.459	.926	.971	.997	1.000
16	.014	.026	.046	.109	.543	.886	.966	.983	.989
18	.003	.014	.031	.074	.426	.894	.931	.983	.989
20	.026	.029	.049	.060	.451	.900	.960	.983	.991

Table C.11

EMPIRICALLY DERIVED POWERS FOR B2,P
(AGAINST BINOMIAL VARIATES)

ALPHA LEVELS

N	.01	.02	.05	.10	.50	.90	.95	.98	.99
					P=2				
4	.009	.011	.043	.100	.509	.697	.954	.989	.994
5	.003	.023	.037	.066	.431	.680	.949	.980	.989
6	.020	.043	.066	.120	.497	.849	.926	.989	.994
7	.011	.029	.060	.114	.494	.677	.926	.966	.974
10	.009	.011	.023	.083	.523	.900	.949	.980	.989
12	.011	.026	.063	.094	.480	.677	.931	.969	.977
14	.011	.014	.043	.069	.457	.869	.940	.983	.993
16	.009	.017	.037	.069	.574	.914	.957	.989	.994
18	.009	.017	.037	.103	.534	.691	.951	.980	.983
20	.003	.011	.017	.066	.583	.911	.949	.989	.994
					P=3				
5	0.000	.014	.043	.123	.583	.903	.937	.983	.991
6	0.000	.017	.051	.109	.469	.869	.960	.969	.986
7	.003	.011	.031	.091	.451	.906	.971	.989	.997
10	.009	.009	.031	.071	.477	.860	.937	.980	.993
12	.014	.029	.074	.120	.491	.894	.931	.977	.983
14	.006	.017	.040	.083	.469	.849	.929	.951	.969
16	.011	.017	.049	.129	.494	.903	.963	.983	.989
18	.006	.017	.029	.077	.480	.683	.943	.969	.980
20	.003	.014	.031	.060	.434	.809	.931	.963	.983
					P=4				
6	.005	.023	.051	.113	.509	.856	.926	.969	.983
7	.006	.014	.063	.114	.529	.906	.934	.960	.986
8	.014	.034	.071	.111	.543	.660	.923	.954	.966
10	.017	.031	.054	.110	.535	.950	.951	.980	.980
12	0.000	.014	.029	.103	.449	.920	.971	.994	.994
14	0.000	.011	.054	.106	.514	.697	.940	.980	.991
16	0.000	.011	.043	.089	.489	.833	.946	.977	.989
18	.009	.026	.069	.117	.520	.908	.943	.983	.994
20	.003	.017	.049	.089	.497	.633	.945	.974	.959
					P=8				
10	.011	.017	.046	.083	.497	.914	.940	.989	.994
12	.003	.006	.020	.069	.503	.894	.951	.980	.986
14	0.000	0.000	.014	.063	.466	.894	.937	.977	.983
16	.020	.026	.071	.134	.494	.891	.917	.969	.977
18	.003	.009	.026	.066	.511	.926	.957	.991	.994
20	.017	.037	.063	.114	.563	.917	.963	.991	.991

Table C.12
EMPIRICALLY DERIVED POWERS FOR THE W* STATISTIC
(AGAINST BINOMIAL VARIATES)

N	ALPHA LEVELS .01	.02	.05	.10	.50	.90	.95	.98	.99
					P=2				
4	.091	.091	.097	.129	.489	.894	.946	.983	.983
5	.046	.057	.083	.106	.537	.911	.957	.980	.986
6	.014	.020	.043	.077	.486	.897	.951	.977	.991
7	.011	.023	.026	.071	.471	.929	.943	.994	.994
10	.006	.006	.023	.074	.466	.926	.951	.983	.991
12	.021	.040	.071	.100	.489	.923	.957	.994	.997
14	.003	.009	.020	.066	.520	.903	.946	.969	.986
16	.005	.009	.029	.100	.500	.886	.917	.969	.977
18	.014	.020	.066	.120	.549	.877	.929	.971	.983
20	.017	.029	.054	.086	.506	.894	.946	.983	.991
					P=3				
5	.046	.049	.057	.097	.546	.926	.940	.960	.983
6	.017	.031	.063	.123	.566	.920	.954	.963	.997
7	.005	.011	.043	.083	.520	.886	.920	.954	.963
10	.009	.017	.046	.120	.523	.897	.957	.971	.989
12	.006	.023	.074	.109	.523	.897	.951	.974	.994
14	.011	.020	.043	.109	.497	.909	.946	.983	.994
16	.011	.023	.049	.094	.494	.897	.954	.966	.980
18	.009	.014	.051	.094	.511	.897	.963	.986	.989
20	0.000	0.000	.023	.063	.454	.903	.951	.989	.991
					P=4				
6	.014	.023	.054	.094	.494	.889	.931	.966	.989
7	.009	.011	.054	.111	.469	.900	.951	.971	.994
8	.006	.023	.060	.111	.523	.894	.949	.977	.985
10	.003	.014	.046	.111	.500	.889	.954	.986	.991
12	.003	.014	.026	.111	.471	.917	.960	.980	.986
14	.011	.029	.069	.129	.514	.869	.929	.989	1.000
16	.006	.037	.060	.117	.554	.920	.954	.971	.977
18	.014	.026	.066	.094	.431	.926	.949	.974	.980
20	.014	.014	.023	.060	.491	.906	.957	.980	.991

APPENDIX D

This appendix contains power estimates for B1,P, B2,P and W* against the mixed alternative distributions discussed in Chapter III for P=3 and 4, N=5,...,20 and α=.01,...,.99.

Table D.1
POWERS FOR B1,P B2,P & W*, AGAINST MIXED VARIATES
UNIFORM=2 EXPONENTIAL=1

EMPIRICALLY DERIVED POWERS FOR B1,P

ALPHA LEVELS

N	.01	.02	.05	.10	.50	.90	.95	.98	.99
					P=3				
5	.003	.011	.034	.114	.500	.906	.946	.983	.989
6	.017	.034	.051	.103	.509	.917	.954	.986	.991
7	.020	.023	.040	.086	.497	.894	.951	.983	.991
10	.009	.034	.091	.154	.523	.934	.966	.977	.994
12	.063	.086	.131	.206	.669	.943	.971	.986	.994
14	.034	.066	.129	.191	.609	.937	.954	.997	1.000
16	.086	.111	.154	.251	.660	.929	.963	.983	.986
18	.126	.171	.229	.326	.789	.963	.986	.997	.997
20	.097	.114	.211	.286	.677	.960	.980	.989	.994

EMPIRICALLY DERIVED POWERS FOR B2,P

ALPHA LEVELS

N	.01	.02	.05	.10	.50	.90	.95	.98	.99
					P=3				
5	0.000	.009	.023	.077	.509	.894	.946	.986	.994
6	0.000	.020	.043	.083	.429	.691	.946	.977	.994
7	.003	.020	.040	.091	.454	.880	.963	.989	.994
10	.011	.011	.043	.089	.517	.894	.949	.986	.989
12	.011	.043	.074	.151	.549	.909	.957	.991	.994
14	.017	.046	.077	.143	.577	.909	.951	.977	.986
16	.040	.054	.134	.194	.577	.917	.960	.991	.994
18	.023	.051	.091	.180	.626	.931	.971	.986	.986
20	.031	.083	.131	.209	.611	.894	.966	.983	.989

EMPIRICALLY DERIVED POWERS FOR THE W* STATISTIC

ALPHA LEVELS

N	.01	.02	.05	.10	.50	.90	.95	.98	.99
					P=3				
5	.020	.029	.083	.157	.509	.897	.920	.963	.977
6	.020	.043	.077	.134	.563	.946	.974	.991	.994
7	.011	.029	.080	.123	.591	.889	.946	.977	.983
10	.057	.094	.160	.254	.594	.920	.963	.986	.994
12	.060	.146	.200	.263	.663	.926	.946	.974	1.000
14	.091	.117	.194	.271	.606	.931	.951	.974	.980
16	.134	.160	.229	.320	.683	.906	.934	.966	.980
18	.106	.177	.306	.411	.723	.934	.960	.989	.989
20	.131	.169	.277	.377	.743	.949	.969	.986	.989

Table D.2
POWERS FOR B1,P, B2,P & W* AGAINST MIXED VARIATES
UNIFORM=2, NORMAL=1

EMPIRICALLY DERIVED POWERS FOR B1,P

ALPHA LEVELS

N	.01	.02	.05	.10	.50	.90	.95	.98	.99
					P=3				
5	0.000	.003	.020	.103	.469	.874	.937	.977	.986
6	.003	.017	.034	.071	.460	.869	.937	.974	.980
7	.009	.009	.031	.063	.380	.854	.929	.971	.986
10	0.000	.003	.011	.029	.374	.854	.911	.963	.991
12	0.000	.009	.011	.060	.386	.843	.926	.954	.960
14	0.000	.003	.020	.054	.346	.803	.874	.960	.966
16	0.000	.006	.009	.029	.323	.774	.877	.934	.954
18	0.000	.003	.006	.051	.343	.803	.880	.971	.991
20	0.000	.003	.011	.023	.274	.791	.891	.929	.949

EMPIRICALLY DERIVED POWERS FOR B2,P

ALPHA LEVELS

N	.01	.02	.05	.10	.50	.90	.95	.98	.99
					P=3				
5	0.000	.009	.034	.083	.540	.677	.920	.969	.980
6	.003	.017	.057	.114	.469	.917	.971	.994	1.000
7	.014	.020	.034	.080	.526	.929	.977	.994	1.000
10	.011	.011	.049	.109	.511	.894	.949	.980	.989
12	.020	.057	.069	.140	.526	.900	.940	.983	.991
14	.014	.037	.080	.146	.583	.894	.946	.960	.969
16	.040	.057	.137	.197	.631	.931	.960	.989	.994
18	.023	.037	.071	.157	.637	.914	.971	.994	.994
20	.043	.106	.146	.257	.669	.937	.971	.989	.991

EMPIRICALLY DERIVED POWERS FOR THE W* STATISTIC

ALPHA LEVELS

N	.01	.02	.05	.10	.50	.90	.95	.98	.99
					P=3				
5	.017	.029	.051	.129	.509	.894	.917	.957	.980
6	.006	.029	.069	.100	.546	.940	.969	.980	.989
7	.006	.011	.051	.086	.486	.886	.931	.977	.986
10	.003	.009	.049	.060	.474	.871	.943	.960	.994
12	0.000	0.000	.034	.066	.434	.863	.946	.980	.997
14	.009	.011	.037	.069	.360	.849	.931	.963	.977
16	.006	.011	.023	.051	.303	.789	.909	.980	.986
18	0.000	.006	.026	.051	.400	.797	.906	.951	.974
20	.003	.011	.017	.034	.371	.880	.929	.971	.986

Table D.3
POWERS FOR B1,P B2,P & W* AGAINST MIXED VARIATES
EXPONENTIAL=2, NORMAL=1

EMPIRICALLY DERIVED POWERS FOR B1,P

ALPHA LEVELS

N	.01	.02	.05	.10	.50	.90	.95	.98	.99
					P=3				
5	.014	.029	.043	.134	.549	.917	.966	.989	.991
6	.017	.034	.060	.129	.560	.934	.969	.989	.991
7	.034	.046	.094	.203	.591	.929	.974	.991	.991
10	.086	.151	.266	.354	.791	.986	.991	.997	1.000
12	.157	.214	.374	.466	.820	.989	.991	1.000	1.000
14	.197	.283	.377	.477	.897	.991	.994	.997	.997
16	.274	.320	.440	.574	.931	.997	1.000	1.000	1.000
18	.357	.429	.554	.689	.934	1.000	1.000	1.000	1.000
20	.400	.483	.657	.709	.951	.997	.997	.997	.997

EMPIRICALLY DERIVED POWERS FOR B2,P

ALPHA LEVELS

N	.01	.02	.05	.10	.50	.90	.95	.98	.99
					P=3				
5	.003	.014	.020	.071	.551	.857	.917	.974	.986
6	.003	.009	.043	.103	.469	.880	.943	.983	.997
7	0.000	.011	.054	.120	.543	.946	.977	.994	.997
10	.040	.063	.117	.169	.614	.903	.957	.931	.994
12	.097	.140	.200	.277	.660	.934	.969	.991	.991
14	.120	.149	.197	.269	.680	.914	.963	.983	.997
16	.171	.177	.260	.380	.726	.954	.986	.997	1.000
18	.126	.217	.346	.466	.734	.940	.971	.986	.989
20	.171	.254	.371	.463	.774	.949	.971	.989	.994

EMPIRICALLY DERIVED POWERS FOR THE W* STATISTIC

ALPHA LEVELS

N	.01	.02	.05	.10	.50	.90	.95	.98	.99
					P=3				
5	.017	.029	.100	.160	.569	.920	.943	.960	.974
6	.026	.060	.114	.180	.611	.929	.960	.989	.994
7	.049	.071	.151	.254	.669	.923	.960	.989	.994
10	.097	.143	.274	.394	.757	.934	.969	.986	.994
12	.114	.231	.317	.386	.783	.957	.980	.999	.994
14	.194	.237	.377	.506	.814	.960	.977	.991	.994
16	.257	.320	.406	.520	.863	.969	.986	.989	.994
18	.206	.314	.509	.629	.883	.980	.994	.994	1.000
20	.363	.431	.551	.663	.917	.980	.991	1.000	1.000

Table D.4
POWERS FOR B1,P B2,P & W* AGAINST MIXED VARIATES
BINOMIAL=2, NORMAL=1

EMPIRICALLY DERIVED POWERS FOR B1,P

ALPHA LEVELS

N	.01	.02	.05	.10	.50	.90	.95	.98	.99
					P=3				
5	.014	.017	.040	.114	.483	.874	.963	.980	.991
6	.017	.034	.051	.063	.483	.931	.966	.991	.997
7	.006	.009	.037	.083	.417	.849	.954	.986	.994
10	.005	.014	.040	.091	.477	.911	.949	.980	.983
12	.011	.020	.057	.114	.469	.906	.946	.971	.974
14	.006	.023	.034	.071	.497	.911	.943	.986	.994
16	.020	.023	.046	.114	.580	.900	.957	.980	.991
18	.014	.026	.046	.094	.517	.883	.923	.994	.994
20	.006	.014	.040	.069	.434	.891	.946	.969	.986

EMPIRICALLY DERIVED POWERS FOR B2,P

ALPHA LEVELS

N	.01	.02	.05	.10	.50	.90	.95	.98	.99
					P=3				
5	0.000	.003	.031	.071	.597	.889	.940	.980	.986
6	.003	.014	.054	.097	.466	.900	.966	.986	.994
7	.011	.014	.034	.074	.471	.917	.974	.980	.994
10	.014	.014	.043	.077	.474	.857	.931	.980	.989
12	.014	.029	.054	.106	.469	.906	.963	.983	.994
14	.014	.014	.037	.071	.463	.886	.929	.969	.977
16	.006	.014	.051	.106	.526	.897	.951	.991	.994
18	.003	.006	.037	.100	.531	.914	.954	.989	.994
20	.006	.023	.034	.074	.437	.820	.926	.957	.963

EMPIRICALLY DERIVED POWERS FOR THE W* STATISTIC

ALPHA LEVELS

N	.01	.02	.05	.10	.50	.90	.95	.98	.99
					P=3				
5	.020	.026	.049	.100	.506	.929	.957	.971	.994
6	.006	.017	.049	.106	.560	.951	.969	.986	.994
7	.006	.011	.057	.097	.534	.937	.954	.969	.983
10	.011	.020	.069	.120	.537	.909	.951	.974	.986
12	.014	.037	.066	.091	.503	.900	.934	.977	.997
14	.009	.009	.040	.103	.477	.923	.966	.980	.986
16	.006	.023	.054	.114	.520	.906	.949	.977	.986
18	0.000	.009	.034	.086	.500	.926	.963	.983	.989
20	.009	.014	.026	.083	.511	.943	.971	.991	.997

Table D.5
POWERS FOR B1,P B2,P & W* AGAINST MIXED VARIATES
BETA=2, NORMAL=1

EMPIRICALLY DERIVED POWERS FOR B1,P

ALPHA LEVELS, P=3

N	.01	.02	.05	.10	.50	.90	.95	.98	.99
5	.011	.014	.040	.140	.517	.903	.954	.971	.986
6	.020	.031	.051	.094	.489	.923	.966	.980	.983
7	.009	.014	.046	.080	.497	.909	.943	.983	.989
10	.023	.051	.091	.143	.569	.934	.969	.986	.994
12	.040	.083	.166	.260	.654	.969	.980	.983	.986
14	.046	.077	.123	.209	.671	.966	.983	1.000	1.000
16	.071	.100	.166	.286	.789	.971	.989	1.000	1.000
18	.089	.134	.189	.326	.786	.986	.994	1.000	1.000
20	.086	.120	.269	.351	.797	.997	1.000	1.000	1.000

EMPIRICALLY DERIVED POWERS FOR B2,P

ALPHA LEVELS, P=3

N	.01	.02	.05	.10	.50	.90	.95	.98	.99
5	0.000	.011	.037	.100	.563	.897	.937	.974	.983
6	0.000	.011	.043	.089	.443	.894	.937	.969	.994
7	.009	.017	.046	.086	.529	.891	.971	.986	.989
10	.006	.020	.057	.109	.514	.871	.934	.980	.989
12	.017	.051	.106	.163	.549	.917	.960	.994	.997
14	.029	.040	.077	.143	.523	.891	.949	.977	.986
16	.023	.037	.103	.140	.557	.931	.980	1.000	1.000
18	.017	.046	.091	.191	.600	.911	.969	.933	.986
20	.011	.031	.100	.169	.577	.891	.937	.974	.986

EMPIRICALLY DERIVED POWERS FOR THE W* STATISTIC

ALPHA LEVELS, P=3

N	.01	.02	.05	.10	.50	.90	.95	.98	.99
5	.009	.011	.063	.123	.517	.897	.923	.957	.989
6	.014	.037	.069	.129	.529	.946	.969	.991	.997
7	.011	.026	.086	.140	.543	.920	.957	.977	.986
10	.023	.029	.097	.149	.631	.954	.974	.989	.997
12	.020	.086	.151	.217	.663	.931	.969	.986	.994
14	.057	.080	.131	.217	.640	.951	.980	.994	.997
16	.037	.080	.171	.274	.726	.940	.963	.989	.994
18	.040	.074	.183	.317	.723	.949	.983	.989	.994
20	.063	.100	.203	.283	.783	.969	.986	.994	.994

Table D.6
POWERS FOR B1,P, B2,P & W* AGAINST MIXED VARIATES
UNIFORM=1, EXPONENTIAL=2

EMPIRICALLY DERIVED POWERS FOR B1,P

ALPHA LEVELS

N	.01	.02	.05	.10	.50	.90	.95	.98	.99
					P=3				
5	.017	.023	.037	.131	.583	.903	.969	.983	.991
6	.034	.046	.091	.151	.646	.951	.980	.997	.997
7	.046	.057	.126	.226	.657	.963	.986	.994	.997
10	.123	.206	.320	.426	.806	.974	.994	.994	.994
12	.303	.374	.483	.580	.883	.980	.994	.997	.997
14	.326	.414	.509	.643	.943	.997	1.000	1.000	1.000
16	.449	.517	.620	.746	.974	1.000	1.000	1.000	1.000
18	.571	.631	.726	.829	.980	1.000	1.000	1.000	1.000
20	.554	.649	.800	.866	.983	1.000	1.000	1.000	1.000

EMPIRICALLY DERIVED POWERS FOR B2,P

ALPHA LEVELS

N	.01	.02	.05	.10	.50	.90	.95	.98	.99
					P=3				
5	.003	.006	.017	.063	.574	.871	.917	.966	.971
6	0.000	.020	.043	.091	.466	.909	.966	.980	.997
7	.003	.020	.069	.134	.557	.926	.977	.989	.994
10	.074	.094	.160	.223	.651	.903	.963	.986	.991
12	.197	.249	.309	.371	.720	.943	.971	.991	.997
14	.186	.240	.329	.406	.751	.940	.966	.983	.989
16	.254	.280	.394	.477	.806	.963	.980	.997	.997
18	.214	.297	.431	.569	.823	.980	.989	.994	.994
20	.240	.297	.497	.571	.797	.946	.974	.980	.983

EMPIRICALLY DERIVED POWERS FOR THE W* STATISTIC

ALPHA LEVELS

N	.01	.02	.05	.10	.50	.90	.95	.98	.99
					P=3				
5	.006	.011	.060	.163	.586	.911	.943	.969	.983
6	.046	.080	.123	.189	.680	.951	.971	.986	.989
7	.031	.071	.183	.266	.663	.937	.966	.974	.980
10	.143	.200	.320	.414	.800	.957	.966	.983	.991
12	.146	.291	.394	.469	.809	.957	.991	.994	1.000
14	.291	.340	.480	.566	.871	.989	.994	.997	.997
16	.343	.420	.526	.669	.914	.983	.989	.994	.997
18	.257	.400	.571	.686	.926	.986	.991	.994	1.000
20	.377	.483	.631	.734	.949	.997	1.000	1.000	1.000

Table D.7

POWERS FOR B1,P B2,P & W* AGAINST MIXED VARIATES
UNIFORM=1, EXPONENTIAL=1, NORMAL=1

EMPIRICALLY DERIVED POWERS FOR B1,P

	ALPHA LEVELS								
N	.01	.02	.05	.10	.50	.90	.95	.98	.99
					P=3				
5	.014	.023	.037	.106	.503	.874	.954	.980	.989
6	.011	.026	.049	.083	.523	.889	.954	.983	.986
7	.014	.020	.046	.100	.520	.894	.943	.974	.980
10	.026	.049	.083	.166	.640	.937	.969	.980	.994
12	.057	.094	.157	.240	.640	.963	.983	.994	.997
14	.074	.117	.174	.249	.700	.946	.977	.991	1.000
16	.083	.114	.154	.251	.717	.963	.983	.994	.997
18	.157	.203	.257	.380	.780	.963	.983	.997	1.000
20	.146	.203	.314	.391	.780	.974	.989	.991	.991

EMPIRICALLY DERIVED POWERS FOR B2,P

	ALPHA LEVELS								
N	.01	.02	.05	.10	.50	.90	.95	.98	.99
					P=3				
5	0.000	.011	.037	.083	.534	.871	.926	.974	.983
6	.006	.014	.046	.129	.469	.909	.969	.994	1.000
7	.006	.014	.046	.091	.511	.923	.966	.983	.991
10	.014	.026	.043	.091	.509	.866	.946	.986	.991
12	.034	.083	.106	.171	.560	.894	.946	.989	.991
14	.037	.040	.089	.146	.540	.869	.920	.969	.983
16	.040	.054	.117	.186	.554	.917	.960	.994	.997
18	.031	.071	.123	.234	.589	.903	.951	.977	.983
20	.051	.089	.177	.277	.631	.897	.954	.969	.977

EMPIRICALLY DERIVED POWERS FOR THE W* STATISTIC

	ALPHA LEVELS								
N	.01	.02	.05	.10	.50	.90	.95	.98	.99
					P=3				
5	.011	.017	.074	.137	.546	.897	.940	.969	.977
6	.017	.046	.080	.137	.574	.931	.960	.991	.997
7	.026	.029	.091	.134	.569	.911	.943	.974	.986
10	.046	.069	.166	.231	.663	.917	.951	.966	.983
12	.066	.114	.171	.237	.631	.914	.960	.989	.997
14	.114	.140	.234	.329	.657	.940	.963	.983	.989
16	.123	.166	.206	.320	.697	.903	.943	.980	.991
18	.117	.194	.320	.411	.777	.963	.989	.994	.994
20	.206	.237	.323	.429	.771	.943	.960	.989	.997

Table D.8

POWERS FOR B1,P B2,P & W* AGAINST MIXED VARIATES
EXPONENTIAL=1, BINOMIAL=1, NORMAL=1

EMPIRICALLY DERIVED POWERS FOR B1,P

ALPHA LEVELS

N	.01	.02	.05	.10	.50	.90	.95	.98	.99
					P=3				
5	.006	.017	.043	.149	.557	.886	.946	.980	.986
6	.023	.031	.063	.100	.526	.943	.963	.971	.983
7	.009	.011	.043	.091	.514	.914	.969	.986	.994
10	.034	.069	.126	.209	.629	.949	.969	.994	.997
12	.054	.106	.191	.294	.677	.966	.997	1.000	1.000
14	.066	.114	.191	.269	.737	.943	.977	.994	.997
16	.137	.157	.251	.363	.803	.974	.991	1.000	1.000
18	.157	.217	.291	.420	.849	.986	.997	1.000	1.000
20	.154	.217	.351	.431	.823	.994	.997	1.000	1.000

EMPIRICALLY DERIVED POWERS FOR B2,P

ALPHA LEVELS

N	.01	.02	.05	.10	.50	.90	.95	.98	.99
					P=3				
5	.003	.014	.037	.123	.609	.926	.949	.986	.994
6	.011	.031	.074	.094	.426	.866	.946	.977	.991
7	.006	.011	.034	.069	.503	.917	.954	.974	.986
10	.011	.031	.054	.129	.560	.856	.957	.983	.989
12	.034	.057	.097	.169	.557	.891	.929	.980	.994
14	.040	.054	.106	.174	.569	.900	.929	.946	.969
16	.080	.091	.163	.203	.614	.929	.966	.994	.997
18	.037	.097	.151	.257	.629	.923	.971	.989	.991
20	.057	.106	.197	.277	.657	.906	.954	.977	.983

EMPIRICALLY DERIVED POWERS FOR THE W* STATISTIC

ALPHA LEVELS

N	.01	.02	.05	.10	.50	.90	.95	.98	.99
					P=3				
5	.026	.029	.071	.129	.546	.903	.963	.974	.994
6	.020	.034	.074	.131	.549	.920	.957	.991	.994
7	.009	.026	.060	.106	.517	.906	.931	.969	.994
10	.071	.091	.169	.260	.643	.937	.974	.986	1.000
12	.057	.126	.191	.263	.669	.937	.966	.986	1.000
14	.126	.151	.240	.343	.703	.951	.969	.980	.989
16	.154	.220	.294	.397	.680	.946	.974	.986	.991
18	.097	.186	.323	.437	.763	.977	.986	.994	.997
20	.191	.243	.337	.449	.774	.963	.986	.997	.997

Table D.9

POWERS FOR B1,P B2,P & W* AGAINST MIXED VARIATES
UNIFORM=1, EXPONENTIAL=1, BINOMIAL=1

EMPIRICALLY DERIVED POWERS FOR B1,P

ALPHA LEVELS, P=3

N	.01	.02	.05	.10	.50	.90	.95	.98	.99
5	.006	.011	.034	.114	.511	.863	.931	.980	.983
6	.014	.017	.049	.089	.531	.937	.974	.991	.994
7	.017	.020	.040	.103	.546	.903	.954	.980	.994
10	.017	.051	.117	.177	.614	.946	.969	.980	.986
12	.066	.100	.169	.271	.686	.934	.974	.991	.991
14	.046	.094	.143	.226	.680	.963	.986	1.000	1.000
16	.097	.123	.194	.266	.769	.963	.977	.986	.997
18	.143	.180	.237	.349	.814	.974	.991	1.000	1.000
20	.149	.183	.294	.383	.774	.960	.986	.997	1.000

EMPIRICALLY DERIVED POWERS FOR B2,P

ALPHA LEVELS, P=3

N	.01	.02	.05	.10	.50	.90	.95	.98	.99
5	0.000	.011	.026	.106	.551	.889	.926	.980	.986
6	.006	.011	.034	.089	.466	.897	.969	.989	.994
7	.003	.020	.037	.089	.483	.897	.977	.994	.997
10	.023	.026	.051	.100	.500	.871	.931	.969	.983
12	.023	.057	.083	.160	.520	.897	.951	.980	.989
14	.023	.031	.054	.140	.591	.900	.943	.971	.986
16	.046	.063	.129	.194	.557	.909	.951	.977	.980
18	.020	.060	.123	.203	.569	.937	.977	.991	.991
20	.049	.083	.154	.240	.620	.906	.954	.983	.991

EMPIRICALLY DERIVED POWERS FOR THE W* STATISTIC

ALPHA LEVELS, P=3

N	.01	.02	.05	.10	.50	.90	.95	.98	.99
5	.014	.026	.066	.174	.523	.894	.943	.960	.977
6	.026	.037	.089	.129	.583	.931	.966	.994	1.000
7	.014	.043	.097	.154	.557	.894	.920	.969	.963
10	.046	.086	.169	.254	.649	.949	.977	.986	.991
12	.071	.146	.200	.260	.666	.920	.946	.971	.994
14	.091	.123	.200	.286	.623	.946	.966	.983	.991
16	.131	.177	.231	.340	.671	.929	.954	.977	.991
18	.114	.189	.306	.411	.760	.951	.960	.989	.991
20	.194	.246	.334	.431	.774	.957	.974	.991	1.000

Table D.10
POWERS FOR B1,P, B2,P & W* AGAINST MIXED VARIATES
UNIFORM=1, BETA=1, EXPONENTIAL=1, BINOMIAL=1

EMPIRICALLY DERIVED POWERS FOR B1,P

ALPHA LEVELS

N	.01	.02	.05	.10	.50	.90	.95	.98	.99
					P=4				
6	.014	.026	.074	.149	.531	.897	.934	.974	.986
7	.005	.009	.063	.123	.506	.926	.966	.983	.983
8	.011	.023	.049	.114	.577	.906	.943	.977	.977
10	.003	.026	.086	.123	.623	.917	.960	.994	.994
12	.020	.049	.117	.220	.677	.937	.963	.991	.991
14	.071	.091	.126	.189	.660	.969	.986	.994	.997
16	.089	.143	.191	.346	.766	.957	.971	.980	.986
18	.074	.146	.249	.343	.806	.980	.994	.997	.997
20	.166	.189	.303	.423	.860	.986	.994	.994	.997

EMPIRICALLY DERIVED POWERS FOR B2,P

ALPHA LEVELS

N	.01	.02	.05	.10	.50	.90	.95	.98	.99
					P=4				
6	.014	.026	.063	.111	.430	.911	.949	.971	.983
7	.003	.009	.029	.077	.414	.900	.946	.977	.986
8	0.000	.011	.060	.097	.474	.871	.926	.960	.971
10	.000	.011	.054	.105	.439	.880	.946	.983	.989
12	.011	.027	.057	.151	.546	.920	.960	.989	.991
14	.026	.037	.069	.114	.514	.943	.963	.989	.994
16	0.000	.031	.097	.169	.563	.923	.960	.989	.991
18	.023	.063	.123	.194	.557	.979	.957	.989	.994
20	.054	.080	.117	.226	.603	.929	.977	.994	.994

EMPIRICALLY DERIVED POWERS FOR THE W* STATISTIC

ALPHA LEVELS

N	.01	.02	.05	.10	.50	.90	.95	.98	.99
					P=4				
6	.017	.023	.037	.080	.480	.906	.943	.966	.991
7	.023	.034	.080	.151	.549	.894	.931	.980	.986
8	.014	.037	.094	.143	.554	.911	.954	.977	.994
10	.017	.043	.149	.214	.620	.926	.974	.986	.994
12	.029	.083	.131	.263	.634	.894	.946	.983	.983
14	.094	.120	.220	.309	.669	.897	.926	.983	.991
16	.063	.154	.240	.337	.694	.917	.951	.983	.986
18	.146	.180	.300	.366	.706	.949	.960	.994	.994
20	.209	.243	.331	.411	.789	.949	.974	.989	.994

APPENDIX E

This appendix contains plots of B1,P, B2,P and W*. Two types of plots are presented. The first type is Power versus N for fixed P and α. The second type is Power versus Alpha for fixed P and N.

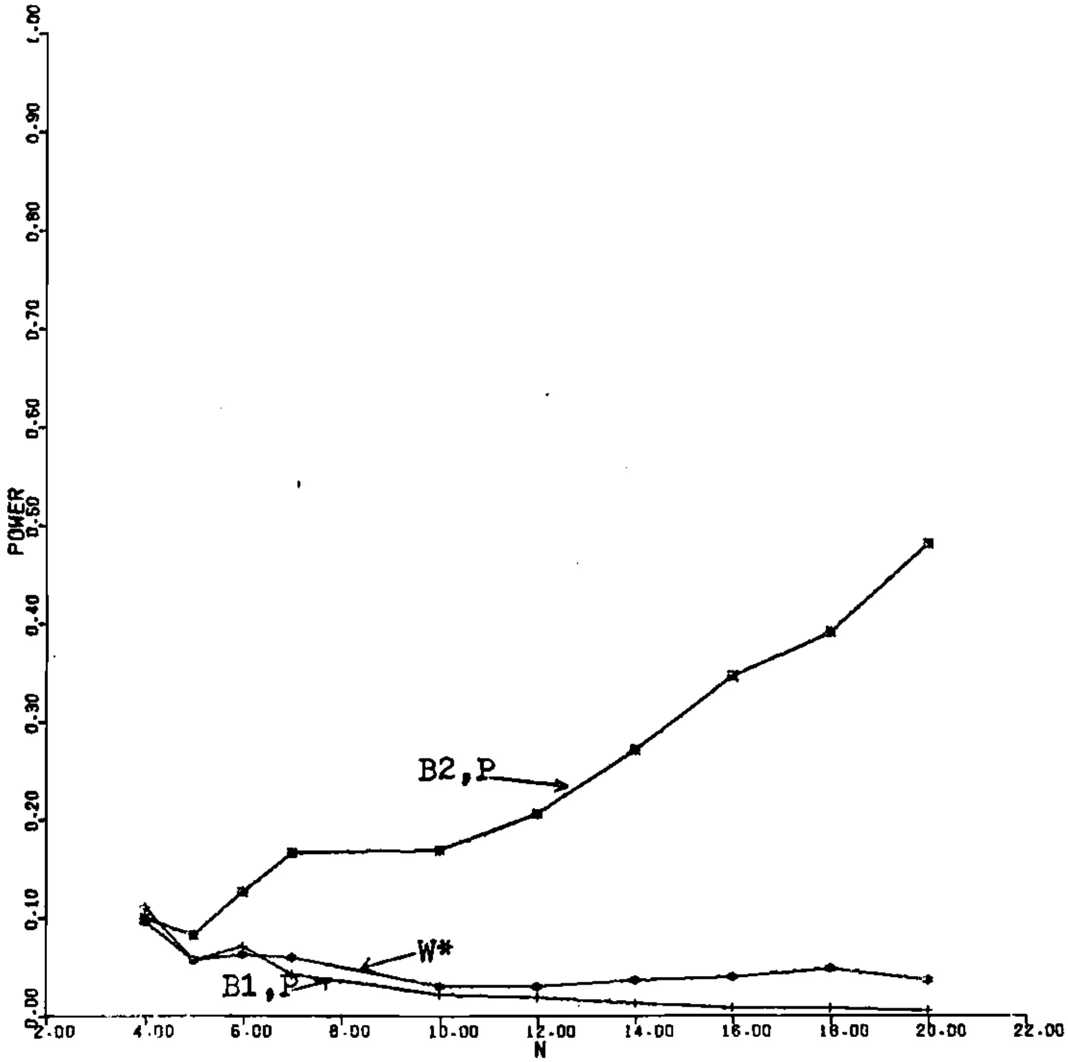

Figure E.1. Plot of B1,P, B2,P and W* Against Uniform Variates, P=2, Alpha=.10

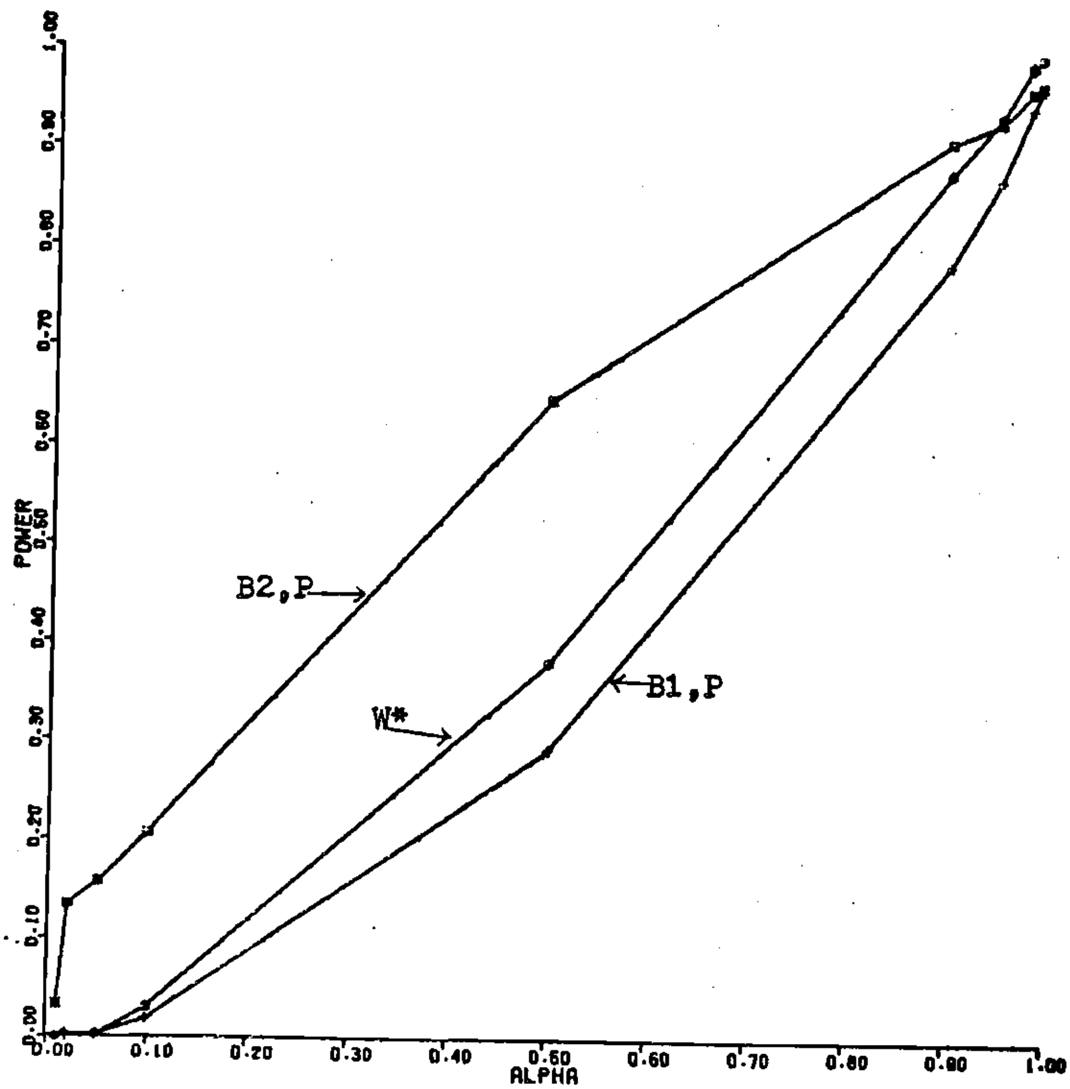

Figure E.2. Plot of B1,P, B2,P and W* Against Uniform Variates, P=2, N=12

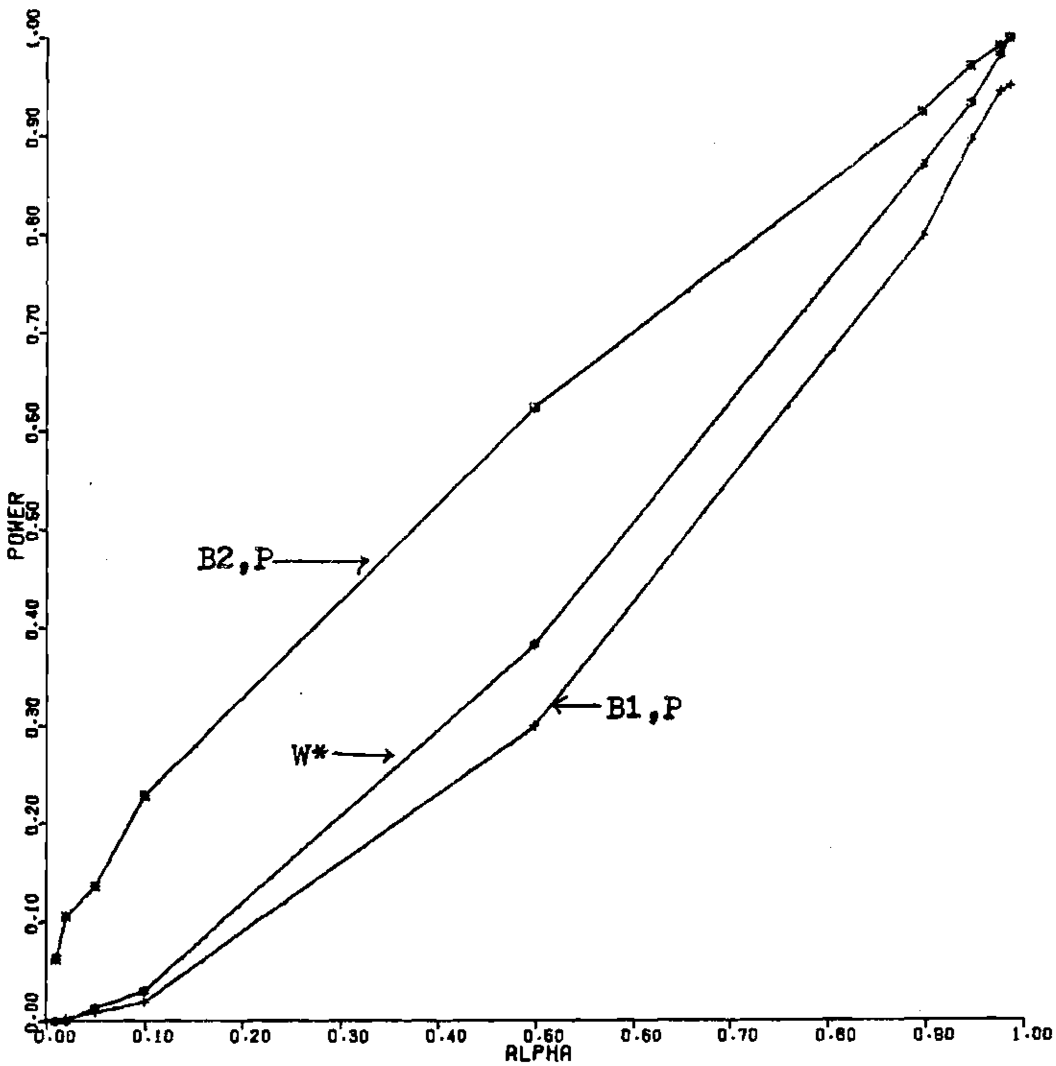

Figure E.3. Plot of B1,P, B2,P and W* Against
Uniform Variates, P=3, N=12

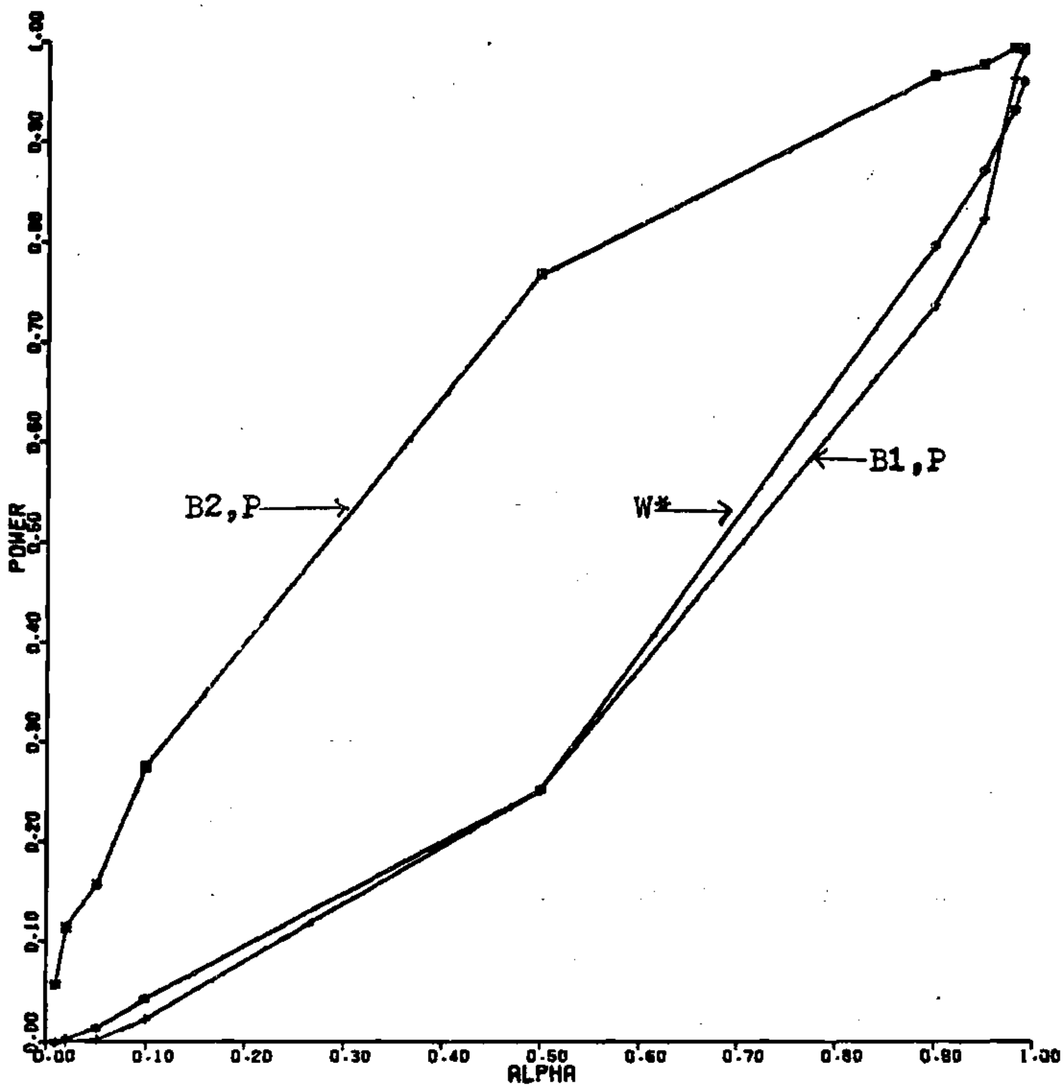

Figure E.4. Plot of B1,P, B2,P and W* Against Uniform Variates, P=3, N=18

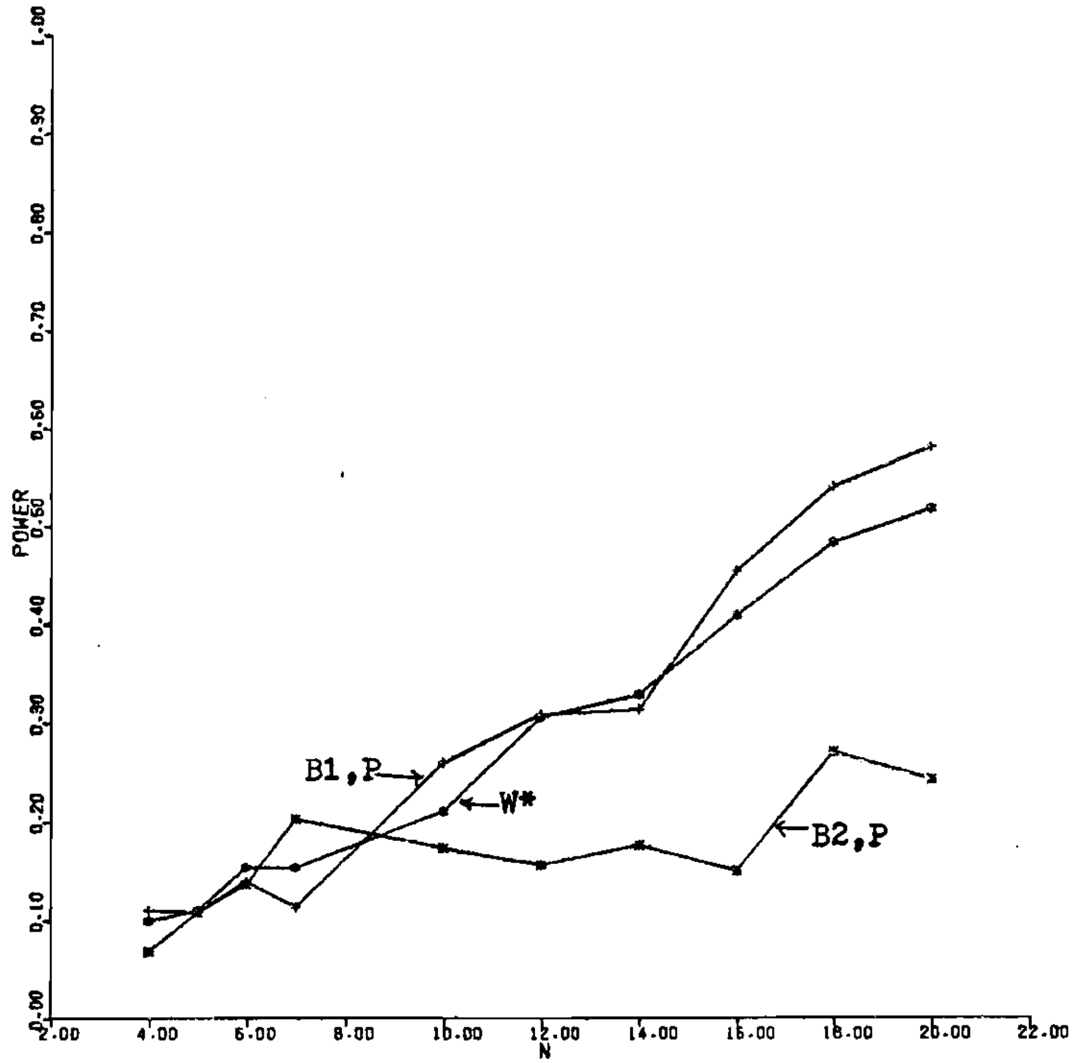

Figure E.5. Plot of B1,P, B2,P and W* Against
Beta Variates, P=2, Alpha=.05

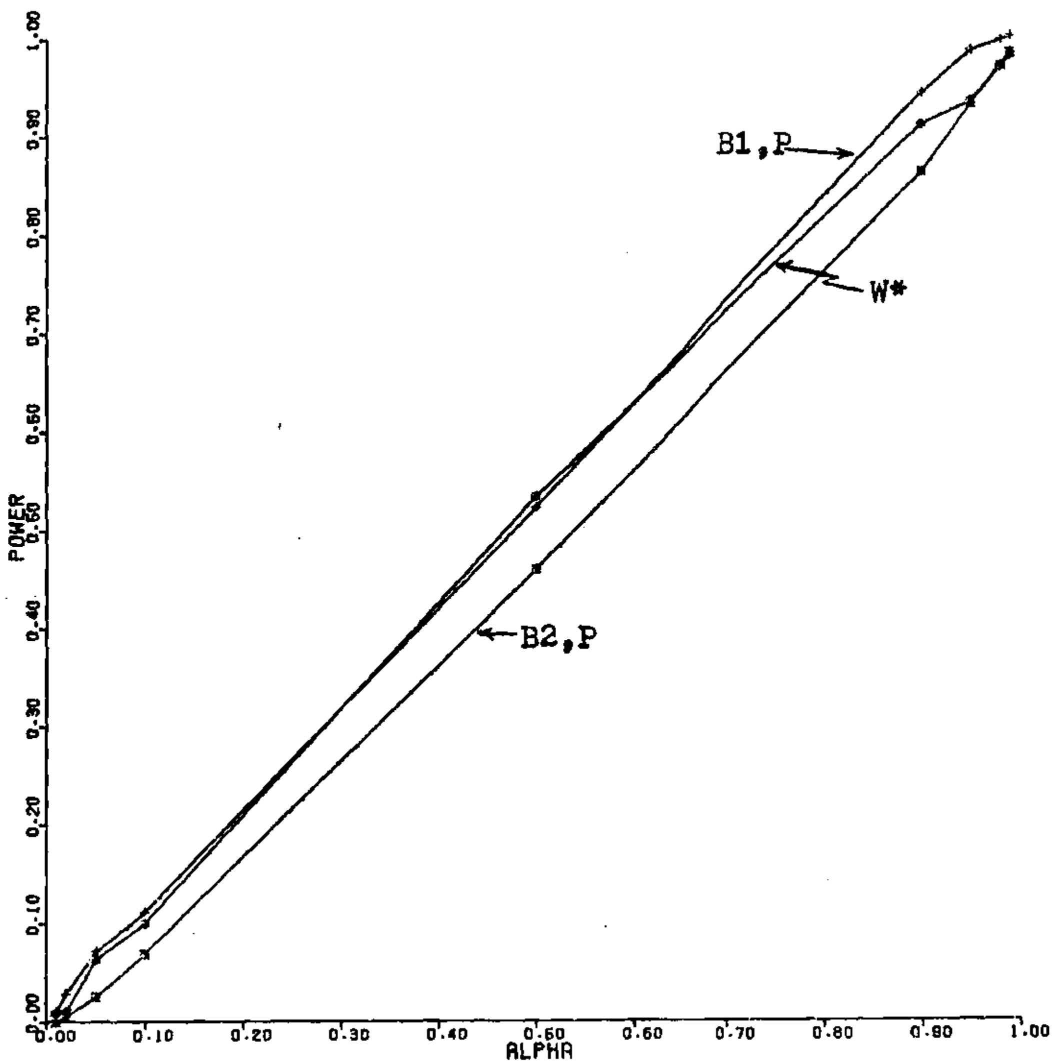

Figure E.6. Plot of B1,P, B2,P and W* Against Beta Variates, P=2, N=4

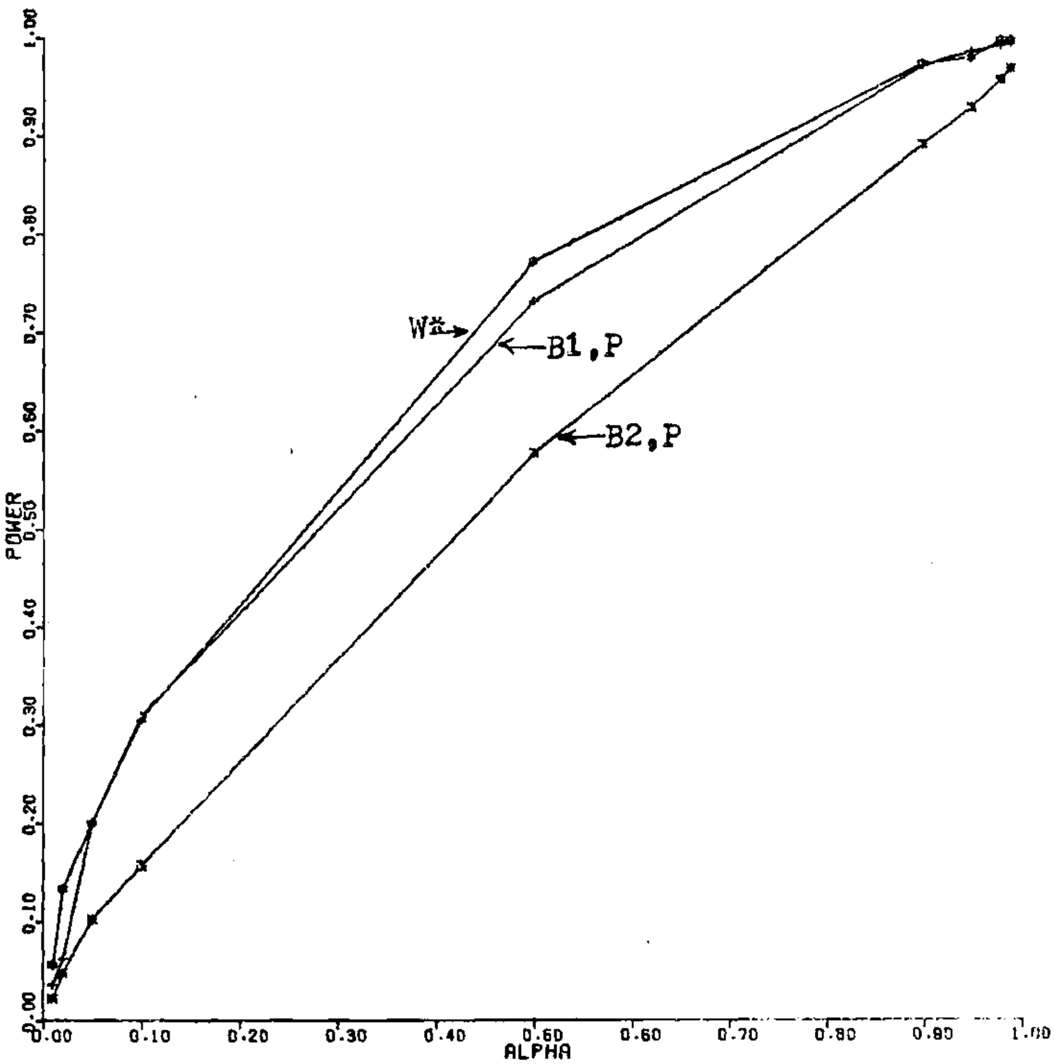

Figure E.7. Plot of B1,P, B2,P and W* Against Beta Variates, P=2, N=12

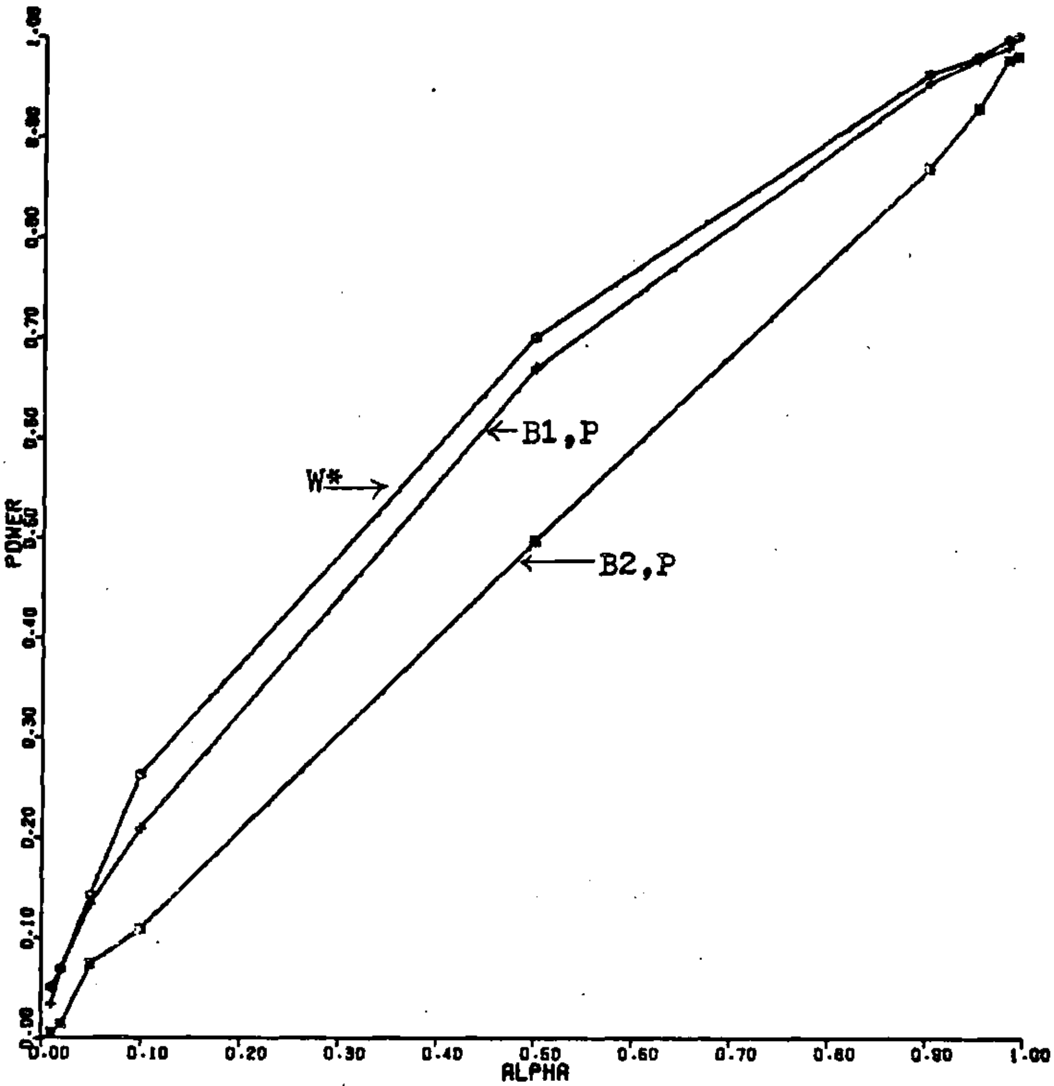

Figure E.8. Plot of B1,P, B2,P and W* Against Beta Variates, P=3, N=10

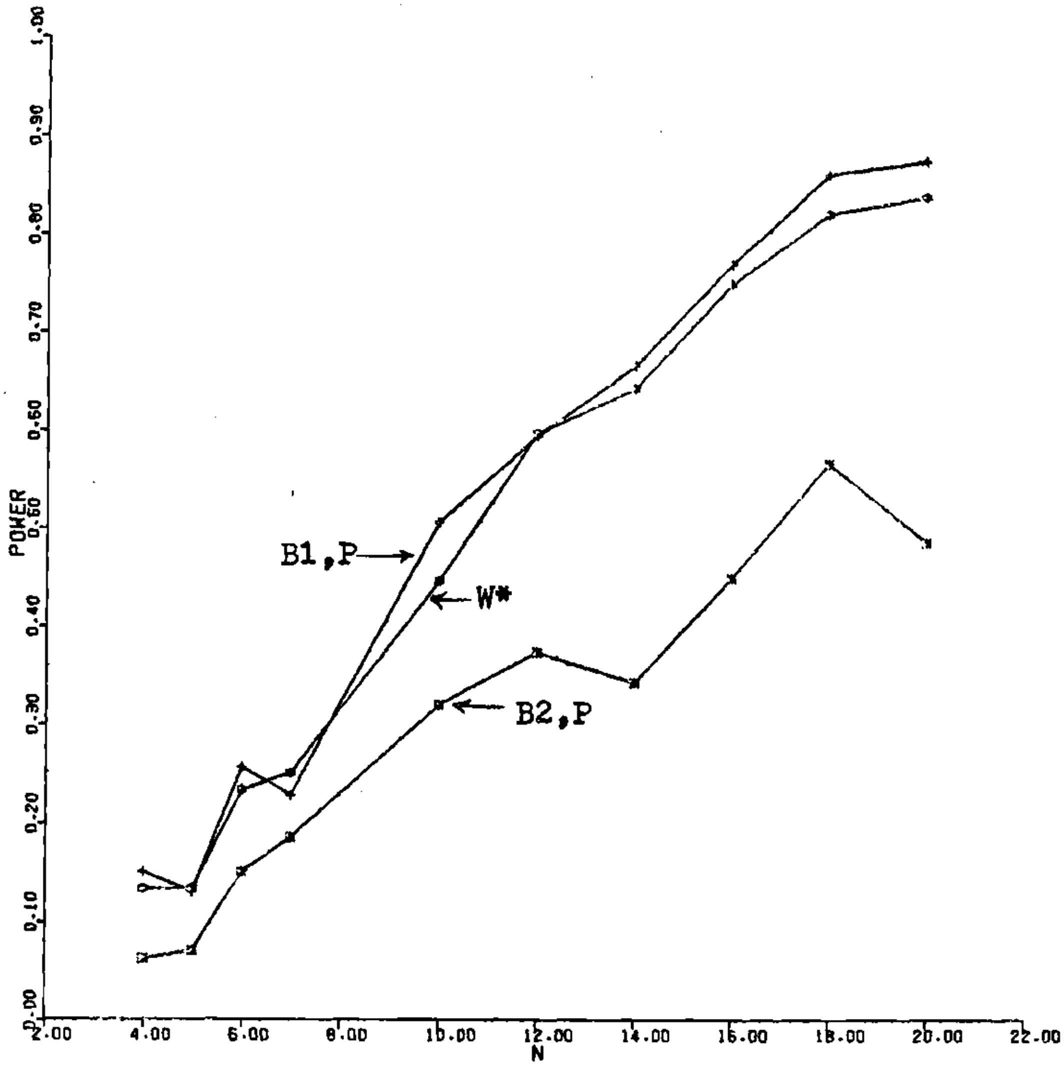

Figure E.9. Plot of B1,P, B2,P and W* Against Exponential Variates, P=2, Alpha=.10

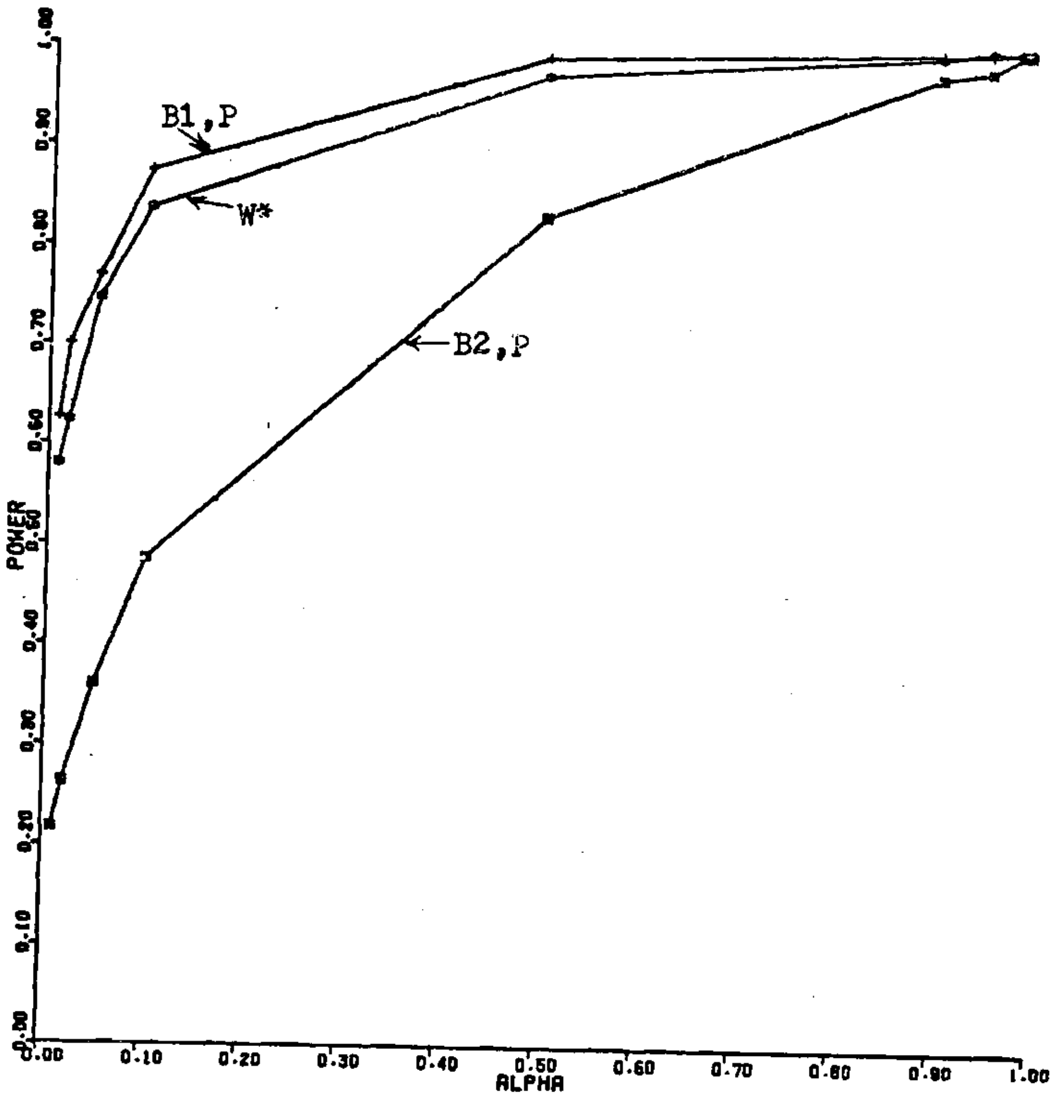

Figure E.10. Plot of B1,P, B2,P and W* Against Exponential Variates, P=2, N=20

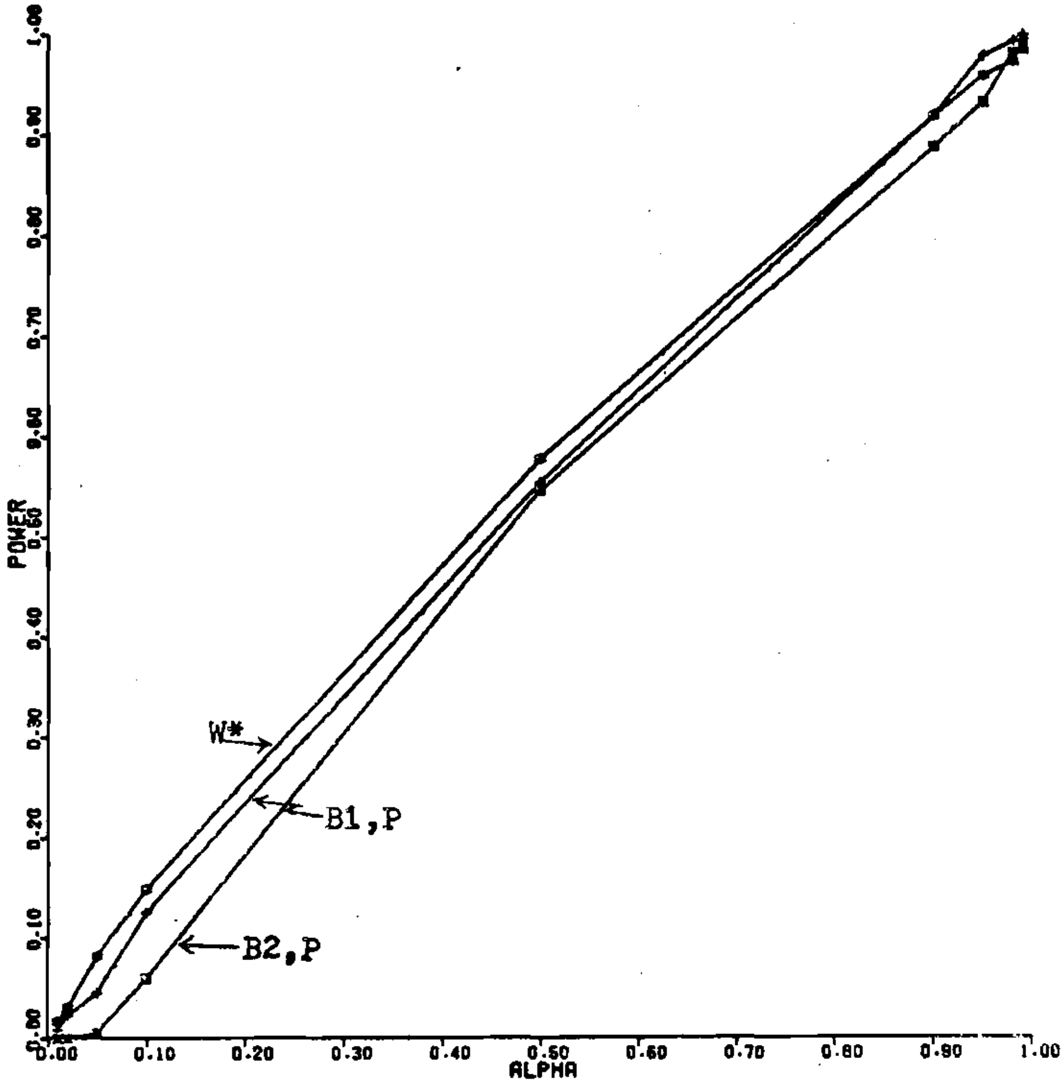

Figure E.11. Plot of B1,P, B2,P and W* Against Exponential Variates, P=3, N=5

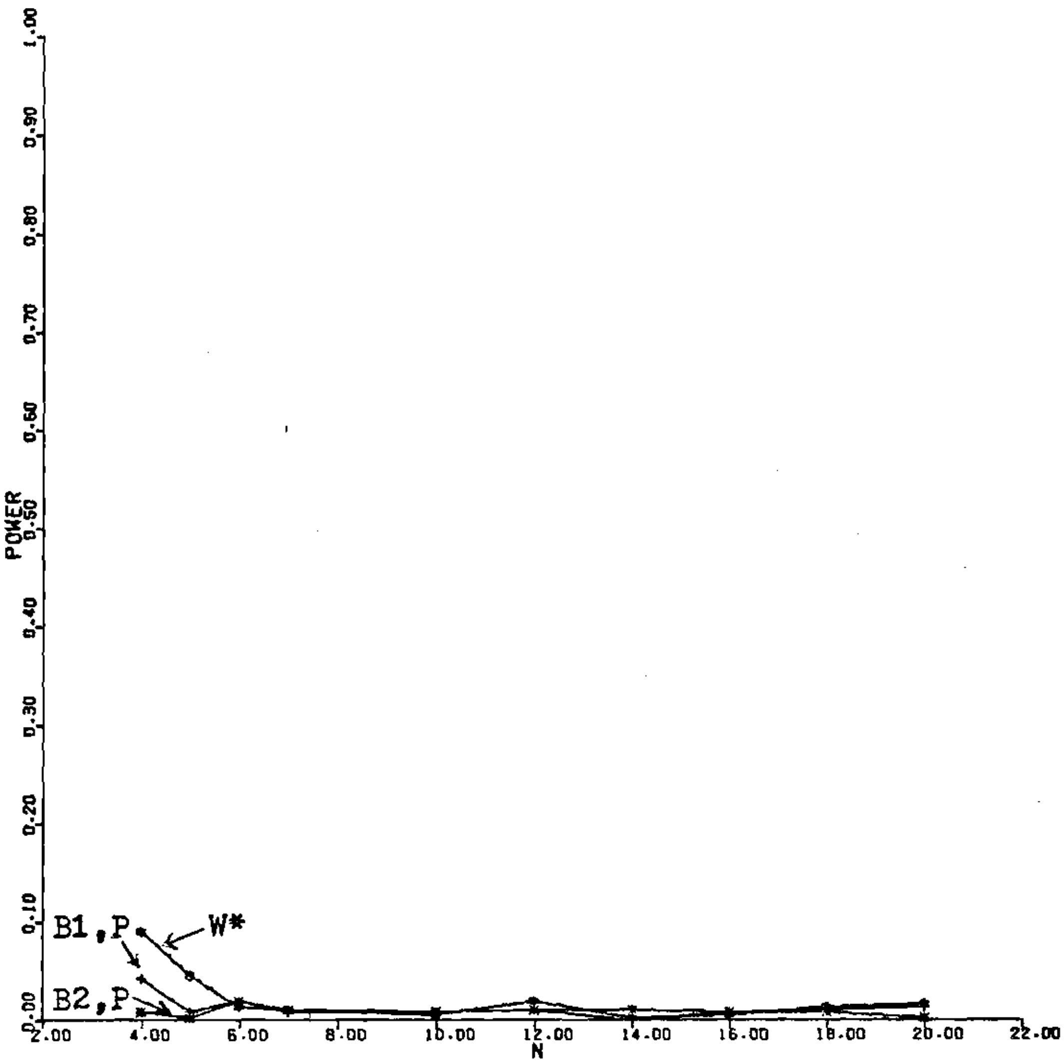

Figure E.12. Plot of B1,P, B2,P and W* Against Binomial Variates, P=2, Alpha=.01

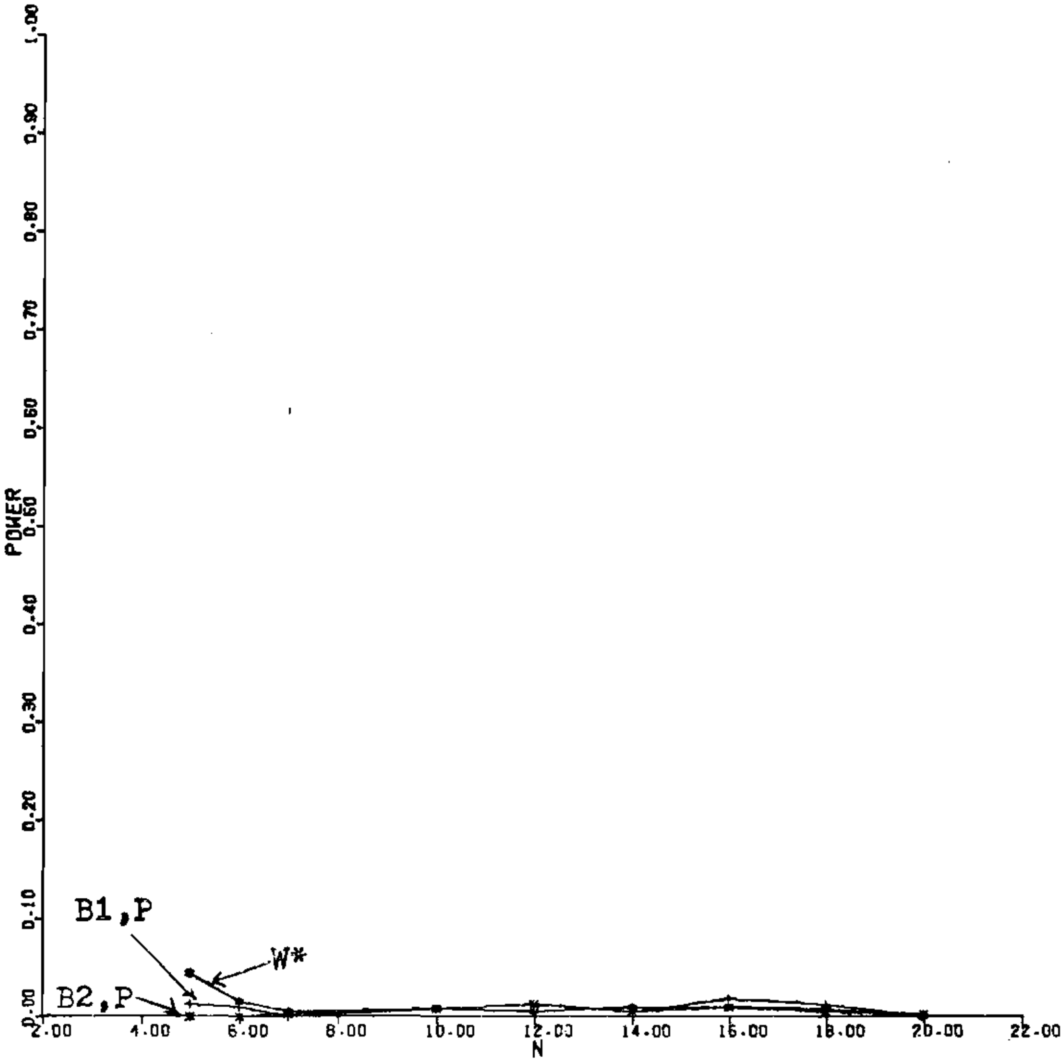

Figure E.13. Plot of B1,P, B2,P and W* Against Binomial Variates, P=3, Alpha=.01

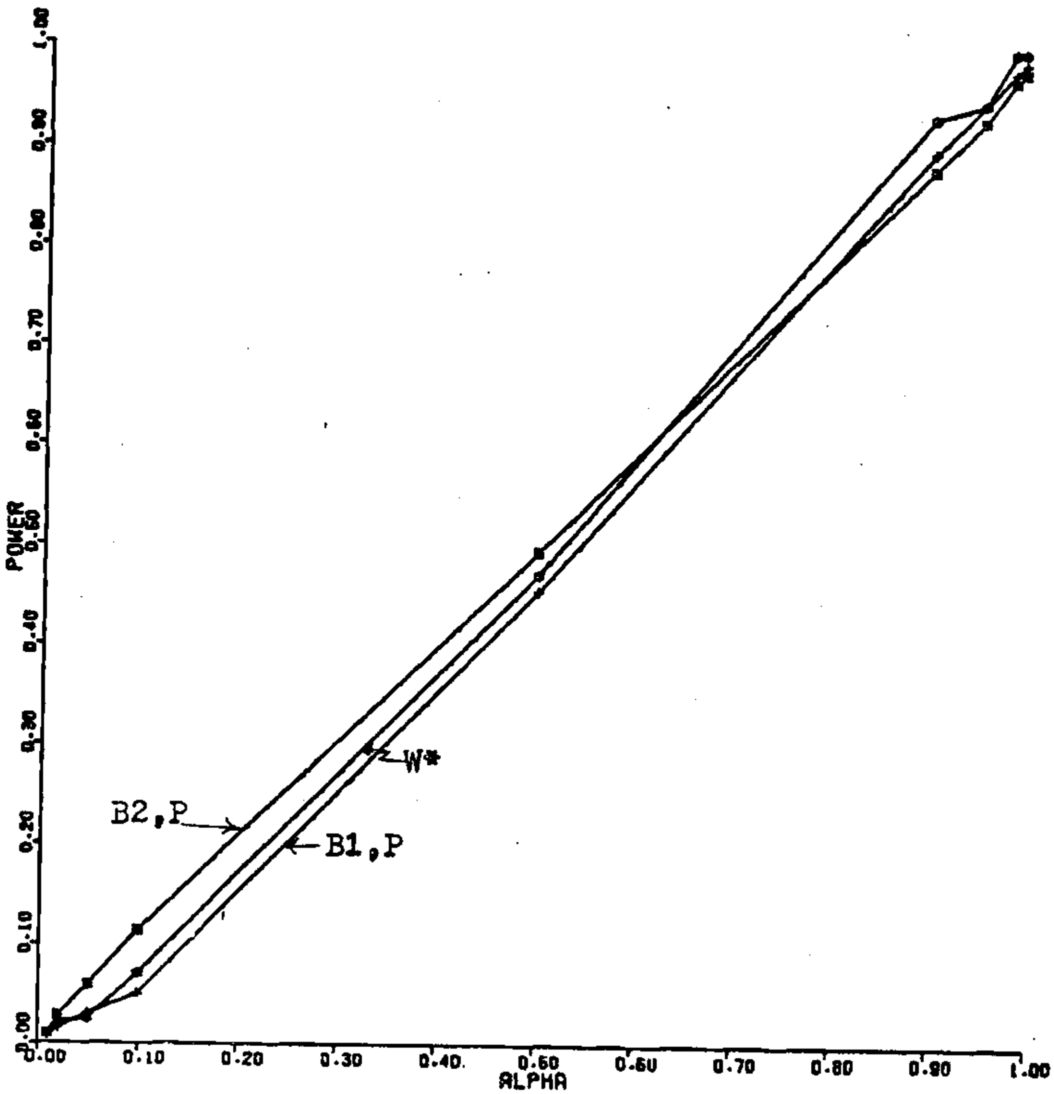

Figure E.14. Plot of B1,P, B2,P and W* Against Binomial Variates, P=2, N=7

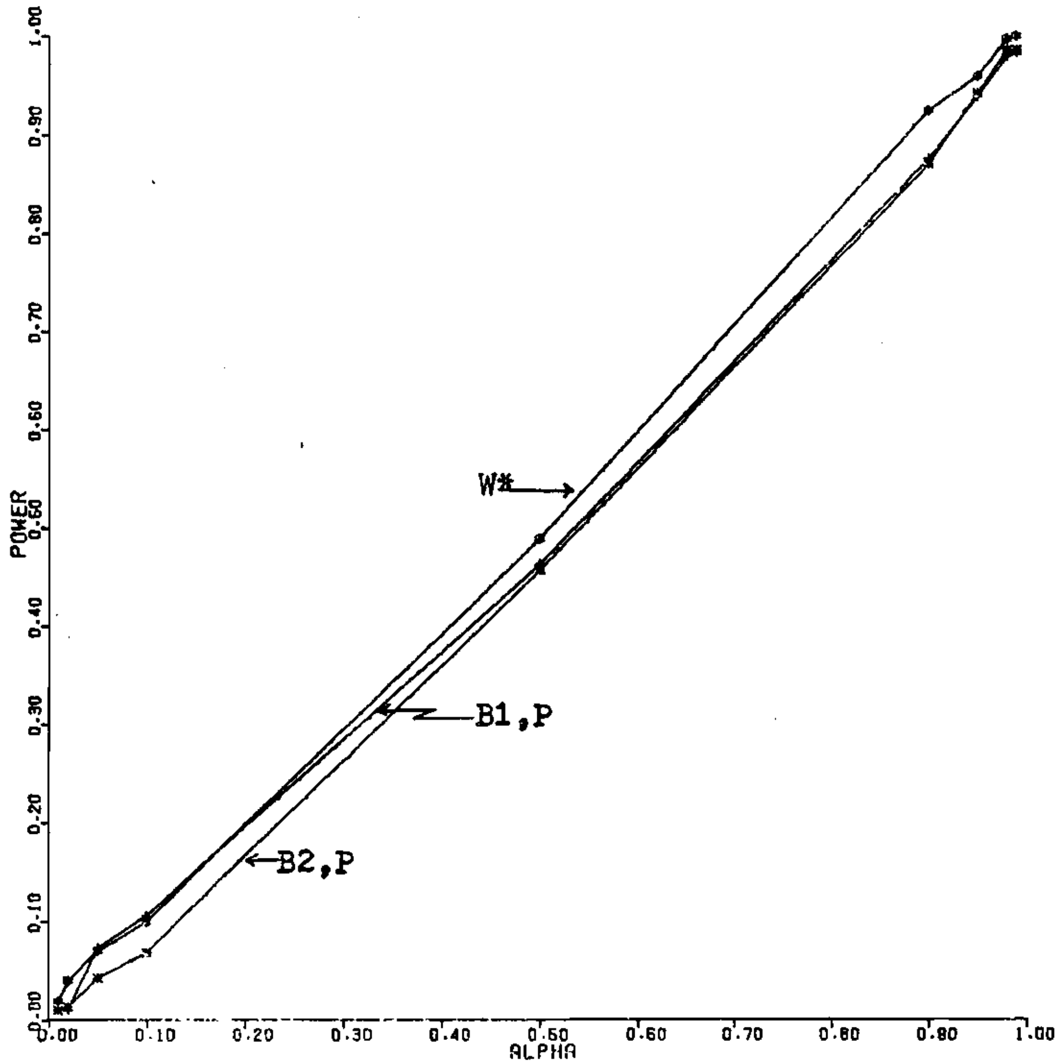

Figure E.15. Plot of B1,P, B2,P and W* Against Binomial Variates, P=2, N=12

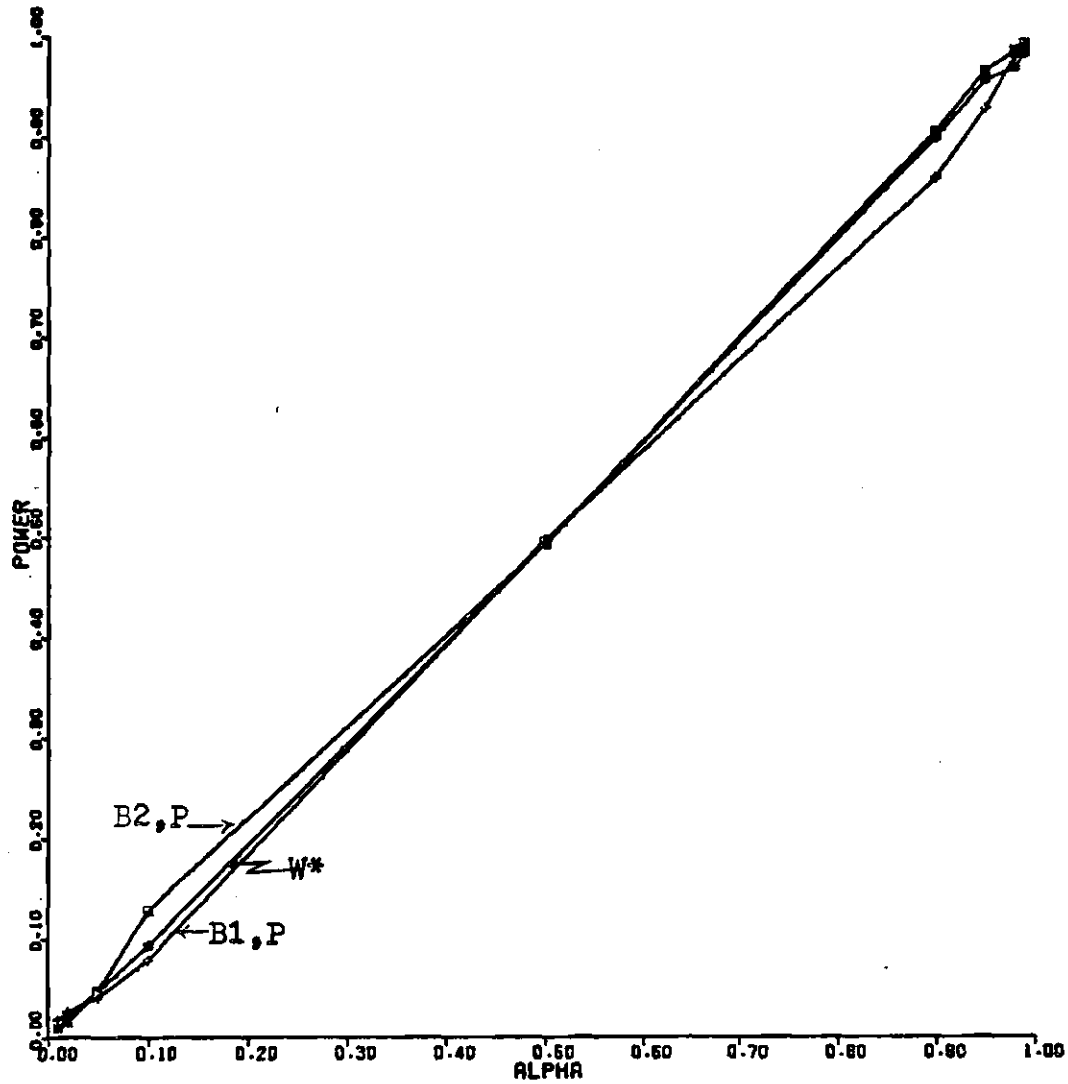

Figure E.16. Plot of B1,P, B2,P and W* Against Binomial Variates, P=3, N=16

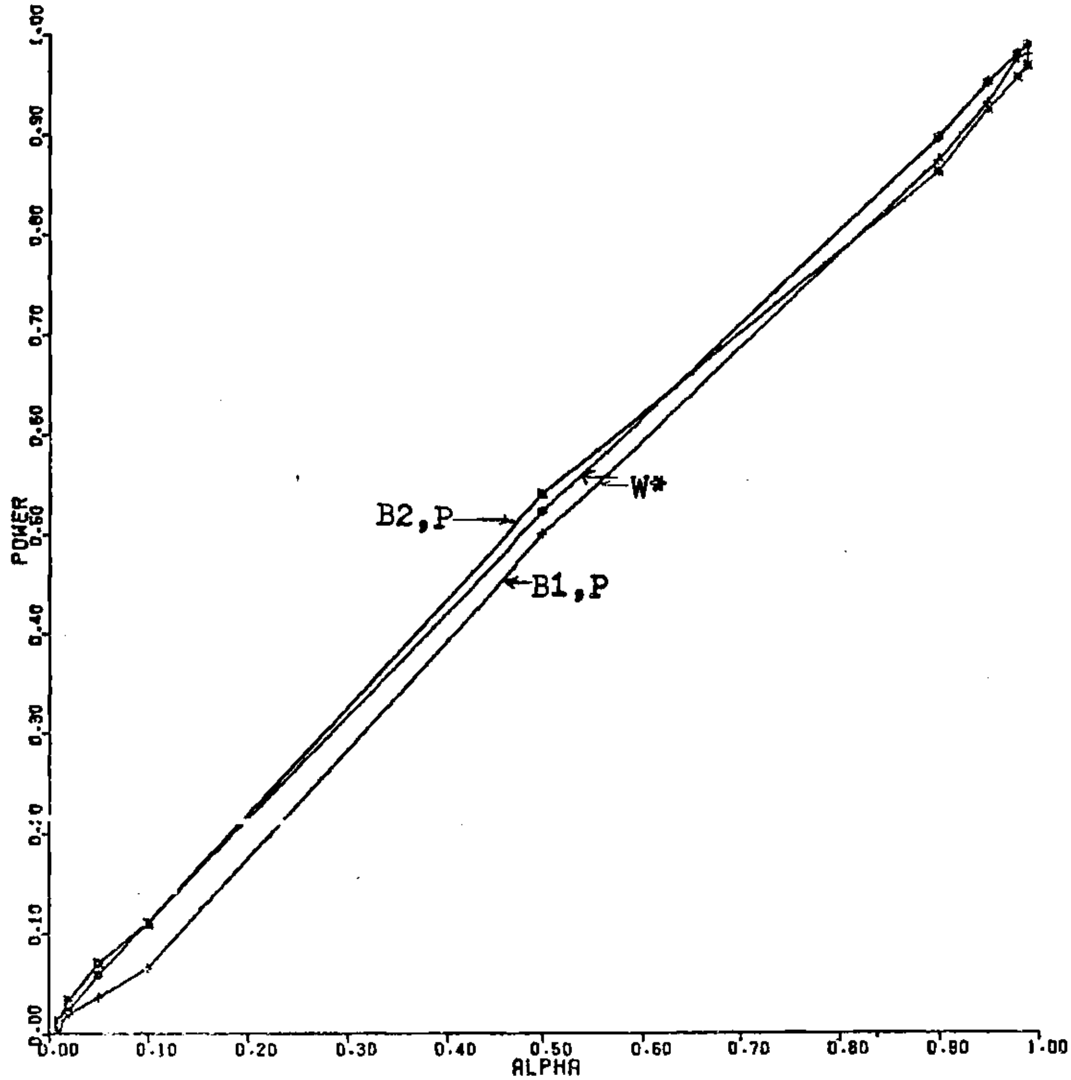

Figure E.17. Plot of B1,P, B2,P and W* Against Binomial Variates, P=4, N=8